WAKE COUNTY PUBLIC SCHOOL SYSTEM

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Transforming Mathematics Education

SECONDARY MATH TWO

An Integrated Approach

WCPSS Math 2 Unit 8: MVP MODULE 6

Similarity & Right Triangle Trigonometry

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The Mathematics Vision Project

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6.1 Photocopy Faux Pas

A Develop Understanding Task

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Burnell has a new job at a copy center helping people use the photocopy machines. Burnell thinks he knows everything about making photocopies, and so he didn't complete his assignment to read the training manual.

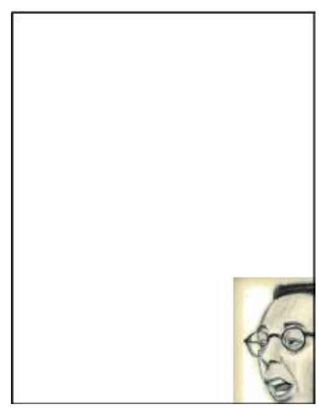
Mr. and Mrs. Donahue are making a scrapbook for Mr. Donahue's grandfather's 75th birthday party, and they want to enlarge a sketch of their grandfather which was drawn when he was in World War II. They have purchased some very expensive scrapbook paper, and they would like this image to be centered on the page. Because they are unfamiliar with the process of enlarging an image, they have come to Burnell for help.

"We would like to make a copy of this image that is twice as big, and centered in the middle of this very expensive scrapbook paper," Mrs. Donahue says. "Can you help us with that?"

"Certainly," says Burnell. "Glad to be of service."

Burnell taped the original image in the middle of a white piece of paper, placed it on the glass of the photocopy machine, inserted the expensive scrapbook paper into the paper tray, and set the enlargement feature at 200%.

In a moment, this image was produced.



"You've ruined our expensive paper," cried Mrs. Donahue. "Much of the image is off the paper instead of being centered."

"And this image is more than twice as big," Mr. Donahue complained. "One fourth of grandpa's picture is taking up as much space as the original."

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In the diagram below, both the original image—which Burnell taped in the middle of a sheet of paper—and the copy of the image have been reproduced in the same figure.

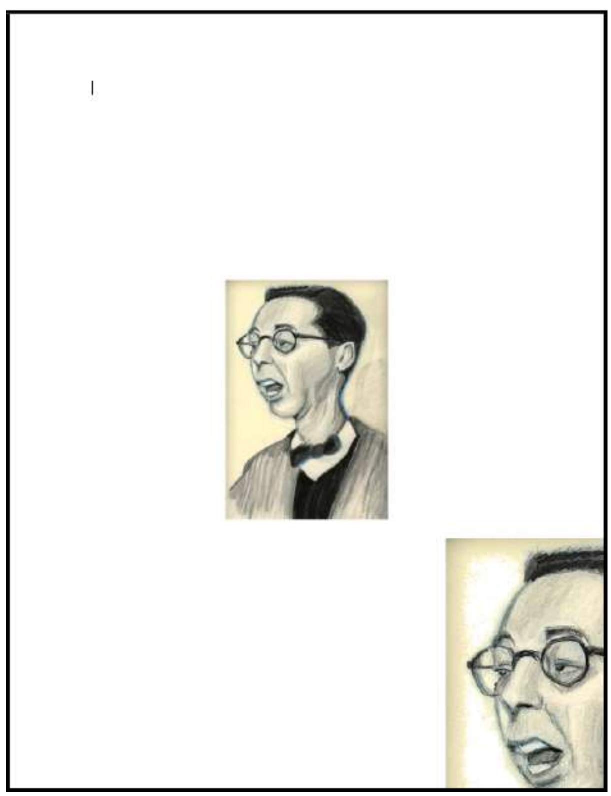
1. Explain how the photocopy machine produced the partial copy of the original image.

2. Using a "rubber band stretcher" finish the rest of the enlarged sketch.

3. Where should Burnell have placed the original image if he wanted the final image to be centered on the paper?

4. Mr. Donahue complained that the copy was four times bigger than the original. What do you think? Did Burnell double the image or quadruple it? What evidence would you use to support your claim?

5. Transforming a figure by shrinking or enlarging it in this way is a called a *dilation*. Based on your thinking about how the photocopy was produced, list all of the things you need to pay attention to when dilating an image.



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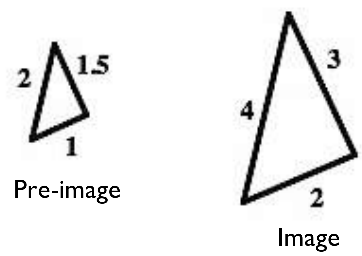
READY, SET, GO! Name Period Date

READY

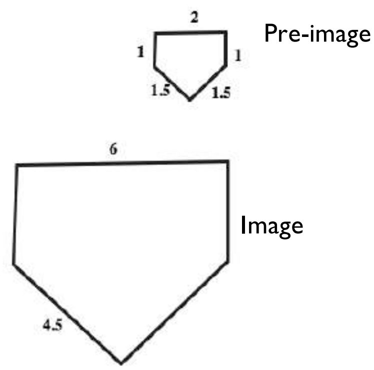
Topic: Scale factors for similar shapes

Give the factor by which each pre-image was multiplied to create the image. Use the scale factor to fill in any missing lengths.

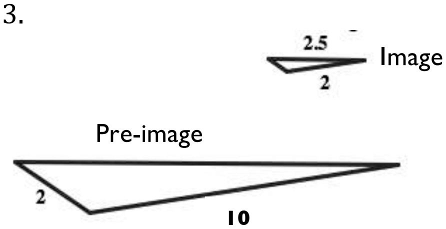
1.



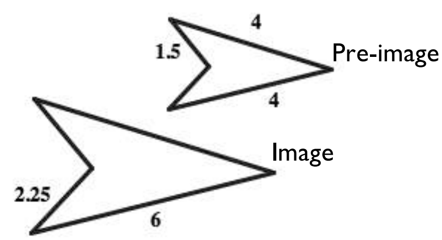
2.



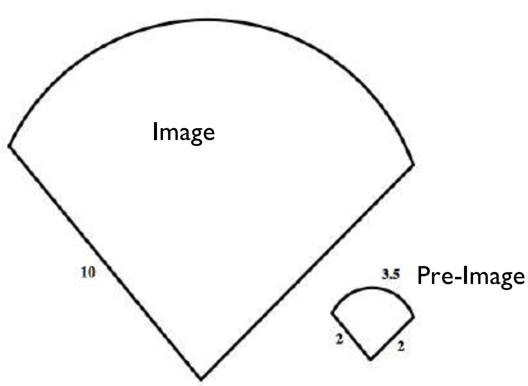
3.



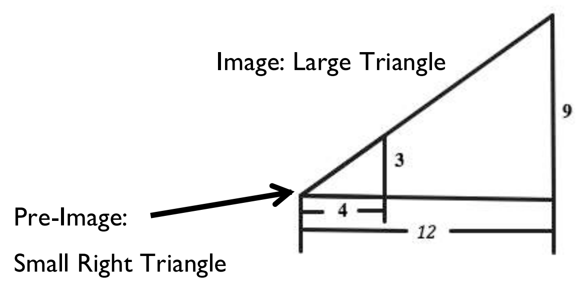
4.



5.



6.



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SET

Topic: Dilations in real world contexts

For each real-world context or circumstance determine the center of the dilation and the tool being used to do the dilation.

7.
Fran walks backward to a distance that will allow her family to all show up in the photo she is about to take.

8.
The theatre technician plays with the zoom in and out buttons in effort to fill the entire movie screen with the image.

9.
Melanie estimates the height of the waterfall by holding out her thumb and using it to see how many thumbs tall to the top of the waterfall from where she is standing. She then uses her thumb to see that a person at the base of the waterfall is half a thumb tall.

10.
A digital animator creates artistic works on her computer. She is currently doing an animation that has several telephone poles along a street that goes off into the distance.

11.
Ms. Sunshine is having her class do a project with a rubber-band tracing device that uses three rubber bands.

12.
A copy machine is set at 300% for making a photo copy.

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GO

Topic: Rates of change related to linear, exponential and quadratic functions

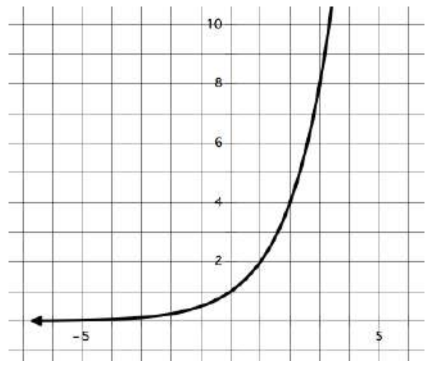
Determine whether the given representation is representative of a linear, exponential or quadratic function, classify as such and justify your reasoning.

13.

X	Y
2	7
3	12
4	19
5	28

Type of function:
Justification:

14.

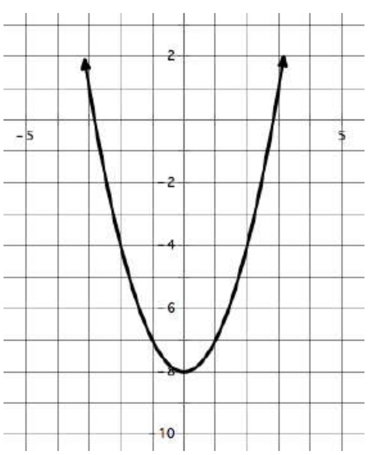


Type of function:
Justification:

15. $y = 3x^2 + 3x$
Type of function:
Justification:

16. $y = 7x - 10$
Type of function:
Justification:

17.



Type of function:

Justification:

18.

X	Y
2	-5
7	5
14	19
25	41

Type of function:

Justification:

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6. 2 Triangle Dilations

A Solidify Understanding Task

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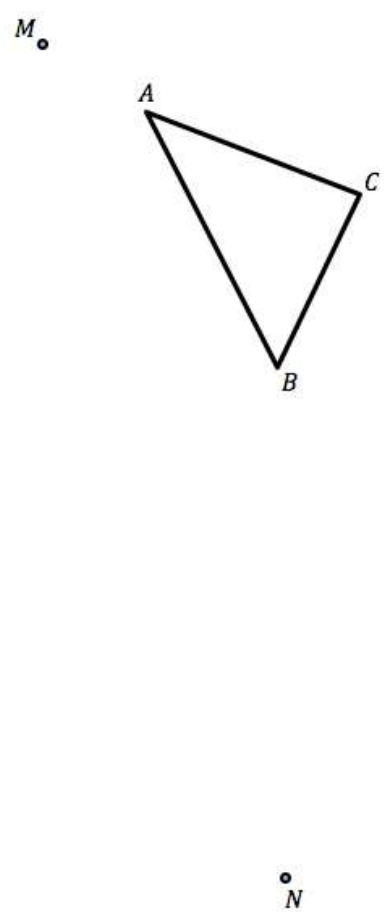
1. Given ΔABC, use point M as the center of a dilation to locate the vertices of a triangle that has side lengths that are three times longer than the sides of ΔABC.

2. Now use point N as the center of a dilation to locate the vertices of a triangle that has side lengths that are one-half the length of the sides of ΔABC.



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3. Label the vertices in the two triangles you created in the diagram above. Based on this diagram, write several proportionality statements you believe are true. First write your proportionality statements using the names of the sides of the triangles in your ratios. Then verify that the proportions are true by replacing the side names with their measurements, measured to the nearest millimeter.

   My list of proportions: (try to find at least 10 proportionality statements you believe are true)

4. Based on your work above, under what conditions are the corresponding line segments in an image and its pre-image parallel after a dilation? That is, which word best completes this statement?

   After a dilation, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.

5. Give reasons for your answer. If you choose "sometimes", be very clear in your explanation about how you can to tell when the corresponding line segments before and after the dilation are parallel and when they are not.

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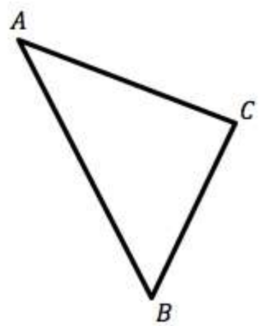
Given ΔABC, use point A as the center of a dilation to locate the vertices of a triangle that has side lengths that are twice as long as the sides of ΔABC.



6. Explain how the diagram you created above can be used to prove the following theorem:

The segment joining midpoints of two sides of a triangle is parallel to the third side and half the length.

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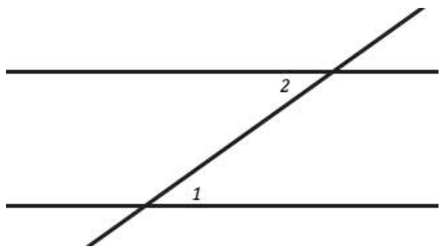
READY, SET, GO! Name Period Date

READY

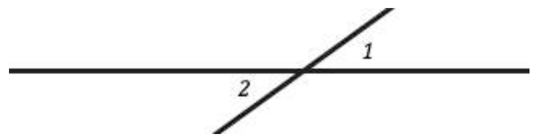
Topic: Basic angle relationships

Match the diagrams below with the best name or phrase that describes the angles.

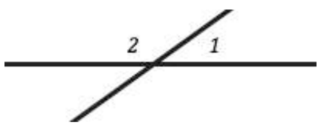
________1.



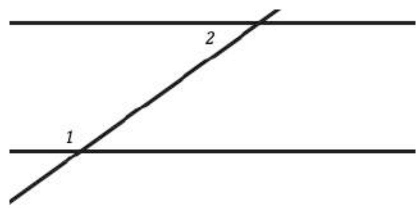
_________2.



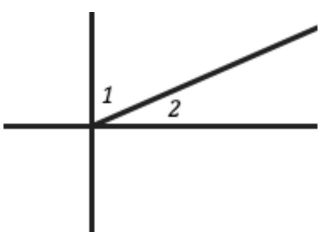
________3.



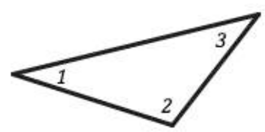
________4.



______5.



________6.



a. Alternate Interior Angles b. Vertical Angles c. Complementary Angles

d. Triangle Sum Theorem e. Linear Pair f. Same Side Interior Angles

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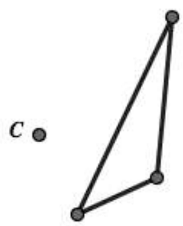
SET

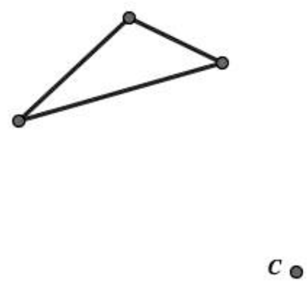
Topic: Performing mathematical dilations and finding the center of dilations.

Use the given pre-image and point *C* as the center of dilation to perform the dilation that is indicated below.

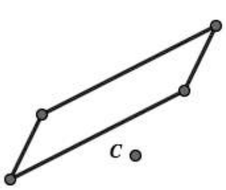
7. Create an image with side lengths twice the size of the given triangle.



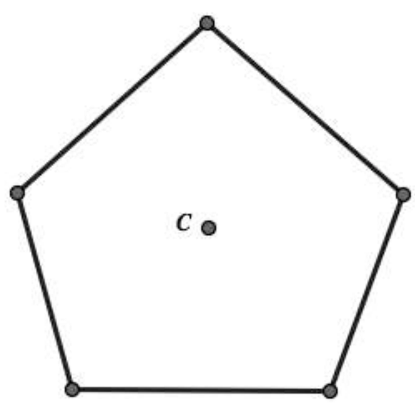
8. Create an image with side lengths half the size of the given triangle.



9. Create an image with side lengths three times the size of the given parallelogram.



10. Create an image with side length one fourth the size of the given pentagon.



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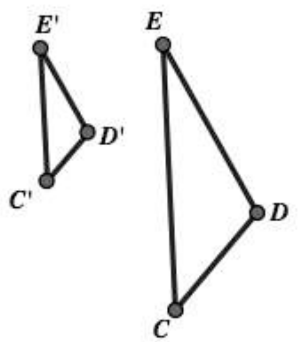
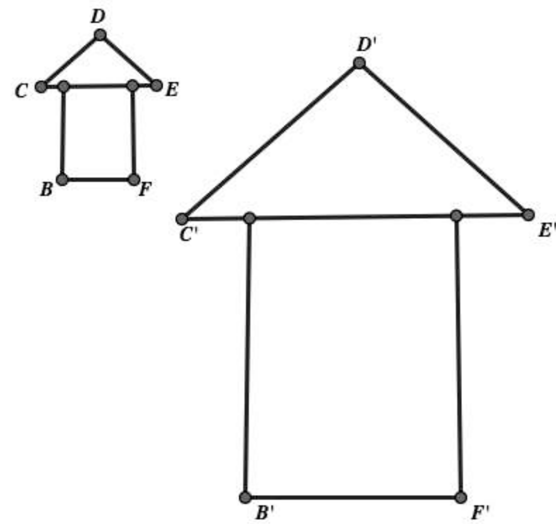
Use the given pre-image and image in each diagram to define the dilation that occurred. Include as many details as possible such as the center of the dilation and the ratio.

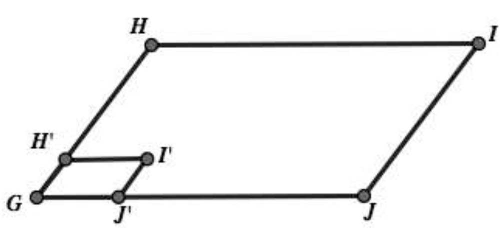
11.



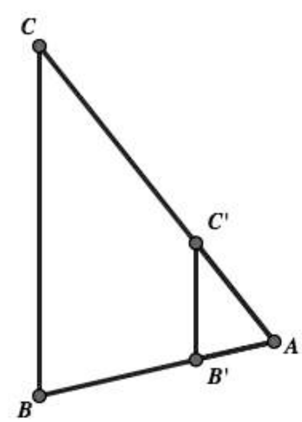
12.



13.



14.



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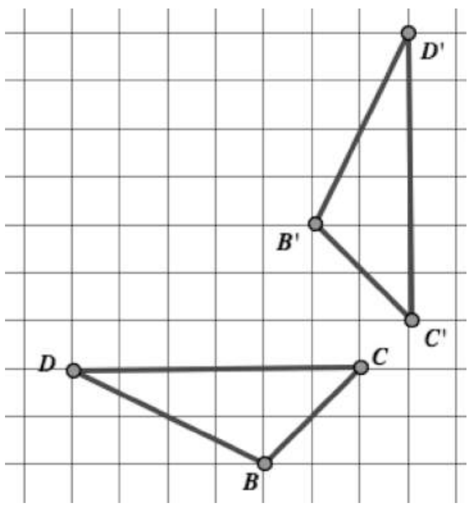
GO

Topic: Classifying mathematical transformations.

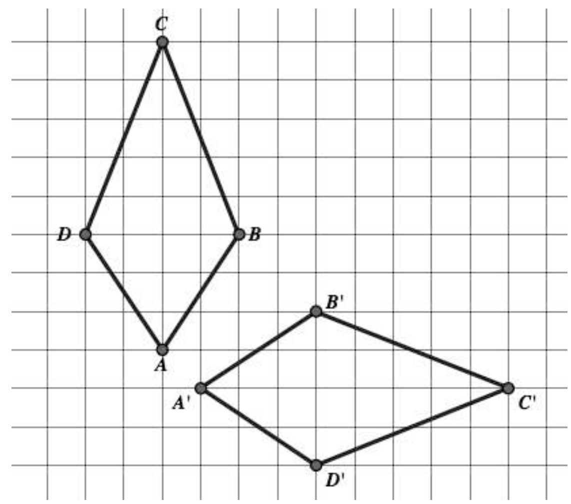
Based on the given image and pre-image determine the transformation that occurred. Further, prove that the transformation occurred by showing evidence of some kind.

(*For example, if the transformation was a reflection show the line of reflection exists and prove that it is the perpendicular bisector of all segments that connect corresponding points from the image and pre-image. Do likewise for rotations, translations and dilations.*)

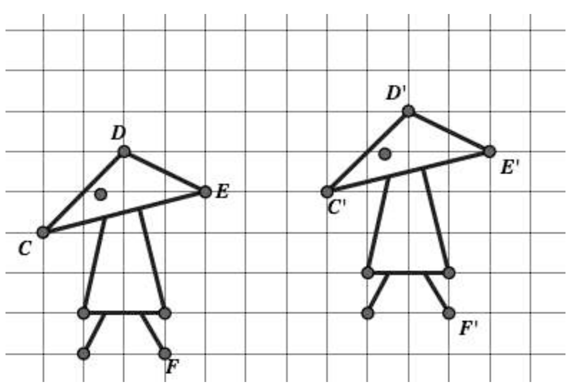
15.



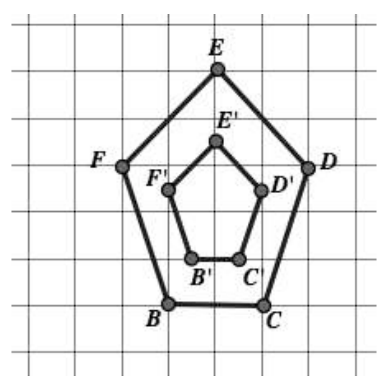
16.



17.



18.



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6. 3 Similar Triangles and Other Figures

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A Solidify Understanding Task

Two figures are said to be *congruent* if the second can be obtained from the first by a sequence of rotations, reflections, and translations. In Mathematics I we found that we only needed three pieces of information to guarantee that two triangles were congruent: SSS, ASA or SAS.

What about AAA? Are two triangles congruent if all three pairs of corresponding angles are congruent? In this task we will consider what is true about such triangles.

Part 1

Definition of Similarity: Two figures are *similar* if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.

Mason and Mia are testing out conjectures about similar polygons. Here is a list of their conjectures.

Conjecture 1: *All rectangles are similar.*

Conjecture 2: *All equilateral triangles are similar.*

Conjecture 3: *All isosceles triangles are similar.*

Conjecture 4: *All rhombuses are similar.*

Conjecture 5: *All squares are similar.*

1. Which of these conjectures do you think are true? Why?

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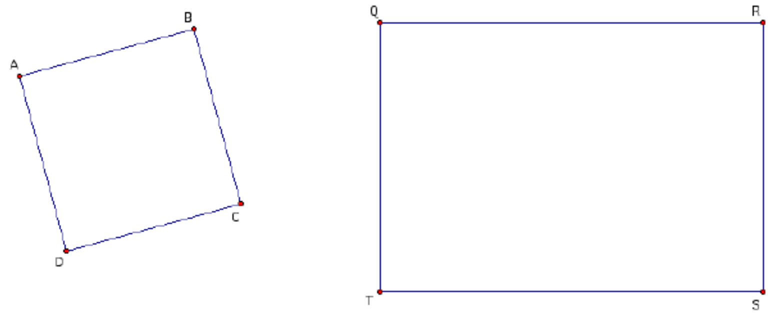
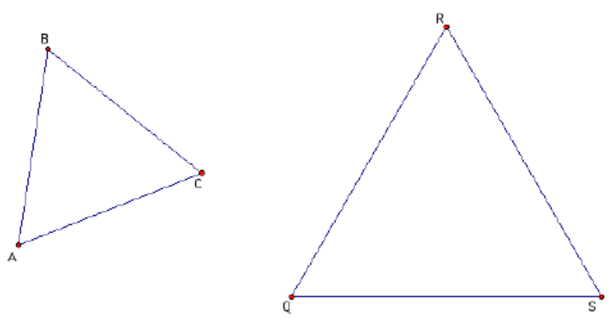
Mason is explaining to Mia why he thinks conjecture 1 is true using the diagram given below.

"All rectangles have four right angles. I can translate and rotate rectangle *ABCD* until vertex *A* coincides with vertex *Q* in rectangle *QRST*. Since $\angle A$ and $\angle Q$ are both right angles, side *AB* will lie on top of side *QR*, and side *AD* will lie on top of side *QT*. I can then dilate rectangle *ABCD* with point *A* as the center of dilation, until points *B*, *C*, and *D* coincide with points *R*, *S*, and *T*.



2. Does Mason's explanation convince you that rectangle *ABCD* is similar to rectangle *QRST* based on the definition of similarity given above? Does his explanation convince you that *all rectangles are similar*? Why or why not?

Mia is explaining to Mason why she thinks conjecture 2 is true using the diagram given below.



"All equilateral triangles have three 60° angles. I can translate and rotate ΔABC until vertex *A* coincides with vertex *Q* in ΔQRS. Since $\angle A$ and $\angle Q$ are both 60° angles, side *AB* will lie on top of side *QR*, and side *AC* will lie on top of side *QS*. I can then dilate ΔABC with point *A* as the center of dilation, until points *B* and *C* coincide with points *R* and *S*."

3. Does Mia's explanation convince you that ΔABC is similar to ΔQRS based on the definition of similarity given above? Does her explanation convince you that *all equilateral triangles are similar*? Why or why not?

4. For each of the other three conjectures, write an argument like Mason's and Mia's to convince someone that the conjecture is true, or explain why you think it is not always true.

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a. Conjecture 3: *All isosceles triangles are similar.*

b. Conjecture 4: *All rhombuses are similar.*

c. Conjecture 5: *All squares are similar.*

While the definition of similarity given at the beginning of the task works for <u>all</u> similar figures, an alternative definition of similarity can be given for polygons: **Two polygons are similar if all corresponding angles are congruent and all corresponding pairs of sides are proportional.**

5. How does this definition help you find the error in Mason's thinking about conjecture 1?

6. How does this definition help confirm Mia's thinking about conjecture 2?

7. How might this definition help you think about the other three conjectures?

   a. Conjecture 3: *All isosceles triangles are similar.*

   b. Conjecture 4: *All rhombuses are similar.*

   c. Conjecture 5: *All squares are similar.*

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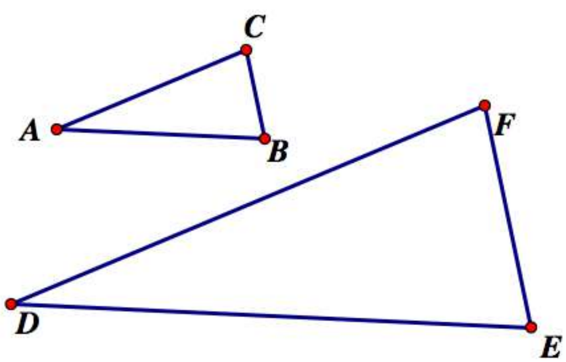
Part 2 (AAA, SAS and SSS Similarity)

From our work above with rectangles it is obvious that knowing that all rectangles have four right angles (an example of AAAA for quadrilaterals) is not enough to claim that all rectangles are similar. What about triangles? In general, are two triangles similar if all three pairs of corresponding angles are congruent?

8. Decide if you think the following conjecture is true.

   Conjecture: *Two triangles are similar if their corresponding angles are congruent.*

9. Explain why you think the conjecture—*two triangles are similar if their corresponding angles are congruent*—is true. Use the following diagram to support your reasoning, Remember to start by marking what you are given to be true (AAA) in the diagram.



Hint: *Begin by translating A to D.*

10. Mia thinks the following conjecture is true. She calls it "*AA Similarity for Triangles*." What do you think? Is it true? Why?

    Conjecture: *Two triangles are similar if they have two pair of corresponding congruent angles.*

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11. Using the diagram given in problem 9, how might you modify your proof that $\Delta ABC \sim \Delta DEF$ if you are given the following information about the two triangles:

a. $\angle A \cong \angle D,\ DE = k \cdot AB,\ DF = k \cdot AC$; that is, $\frac{DE}{AB} = \frac{DF}{AC}$

b. $DE = k \cdot AB,\ DF = k \cdot AC$ and $EF = k \cdot BC$; that is, $\frac{DE}{AB} = \frac{DF}{AC} = \frac{EF}{BC}$

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READY, SET, GO! Name Period Date

READY

Topic: Solving proportions in multiple ways

Solve each proportion. Show your work and check your solution.

1. $\frac{3}{4}=\frac{x}{20}$

2. $\frac{x}{7}=\frac{18}{21}$

3. $\frac{3}{6}=\frac{8}{x}$

4. $\frac{9}{c}=\frac{6}{10}$

5. $\frac{3}{4}=\frac{b+3}{20}$

6. $\frac{7}{12}=\frac{a}{24}$

7. $\frac{a}{2}=\frac{13}{20}$

8. $\frac{3}{b+2}=\frac{6}{5}$

9. $\frac{\sqrt{3}}{2}=\frac{\sqrt{12}}{c}$

SET

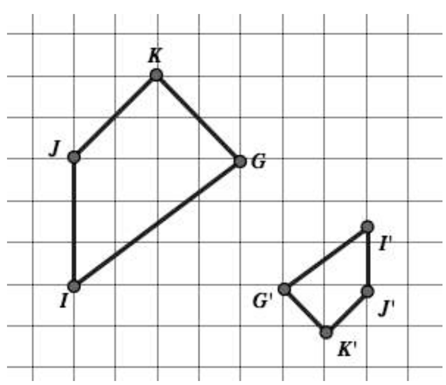
Topic: Proving Shapes are similar

Provide an argument to prove each conjecture, or provide a counterexample to disprove it.

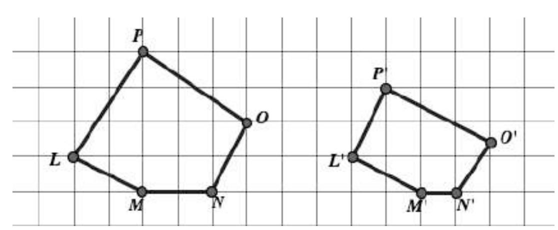
10. All right triangles are similar

11. All regular polygons are similar to other regular polygons with the same number of sides.

12. The polygons on the grid below are similar.



13. The polygons on the grid below are similar.



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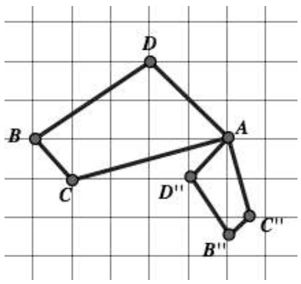
A sequence of transformations occurred to create the two similar polygons. Provide a specific set of steps that can be used to create the image from the pre-image.

14.



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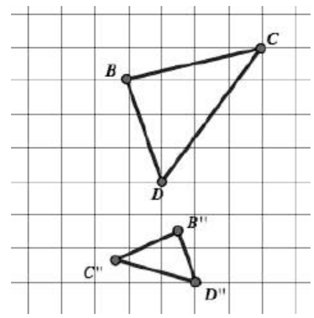

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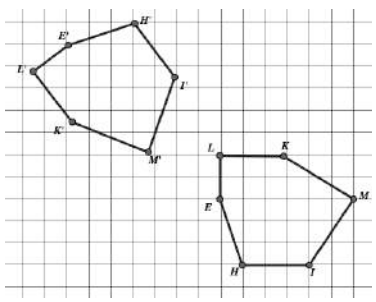

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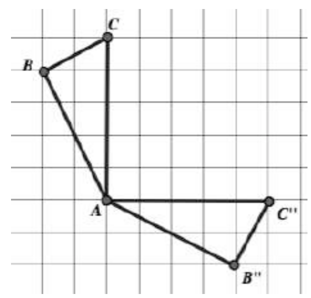
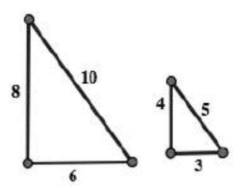


GO

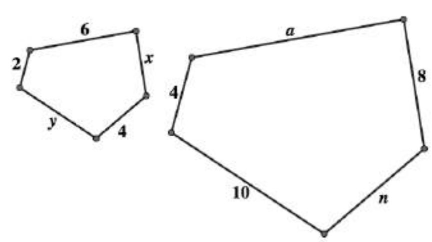
Topic: Ratios in similar polygons

For each pair of similar polygons give three ratios that would be equivalent.

18.



19.



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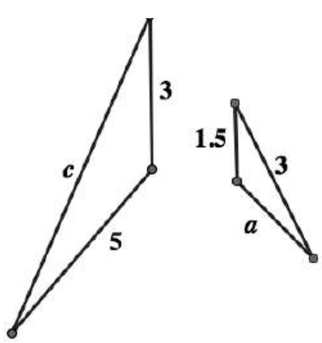

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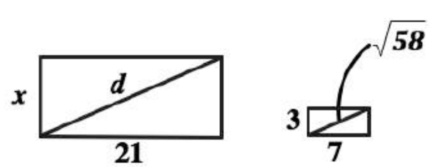

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6. 4 Cut by a Transversal

A Solidify Understanding Task

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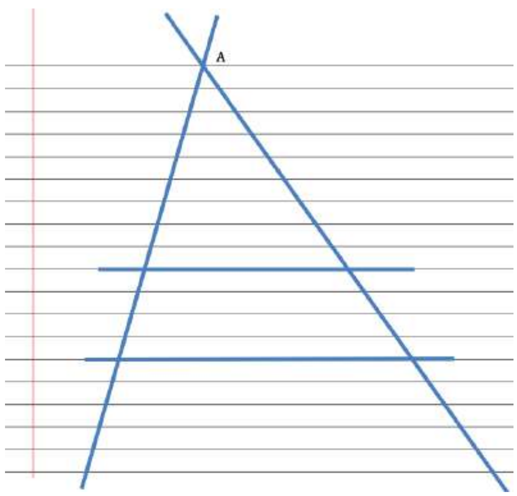
Draw two intersecting transversals on a sheet of lined paper, as in the following diagram. Label the point of intersection of the transversals *A.* Select any two of the horizontal lines to form the third side of two different triangles.



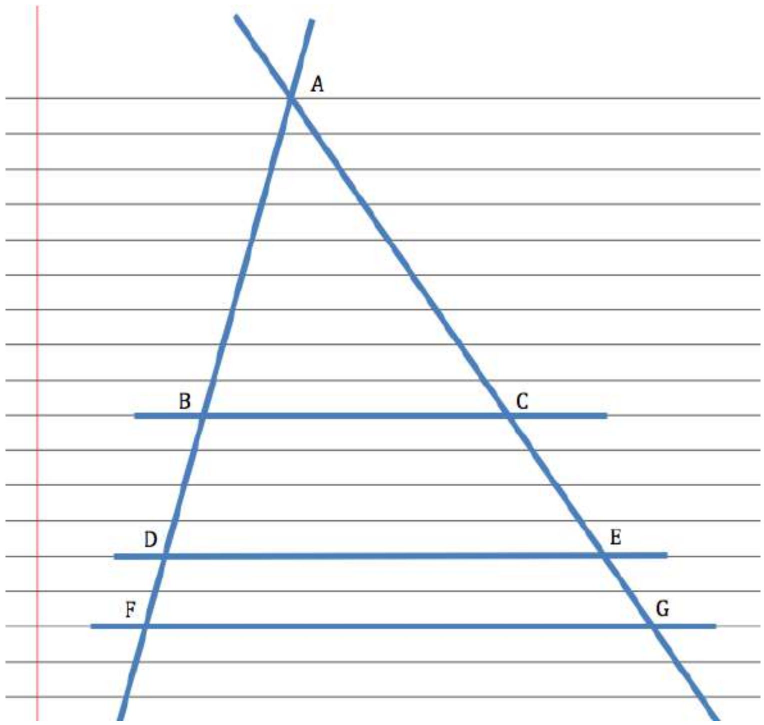
1. What convinces you that the two triangles formed by the transversals and the horizontal lines are similar?

2. Label the vertices of the triangles. Write some proportionality statements about the sides of the triangles and then verify the proportionality statements by measuring the sides of the triangles.

3. Select a third horizontal line segment to form a third triangle that is similar to the other two. Write some additional proportionality statements and verify them with measurements.

Tristan has written this proportion for question 3, based on his diagram below: $\frac{BD}{AB} = \frac{CE}{AC}$

Tia thinks Tristan's proportion is wrong, because some of the segments in his proportion are not sides of a triangle.



4. Check out Tristan's idea using measurements of the segments in his diagram at the left.

5. Now check out this same idea using proportions of segments from your own diagram. Test at least two different proportions, including segments that do not have A as one of their endpoints.

6. Based on your examples, do you think Tristan or Tia is correct?

Tia still isn't convinced, since Tristan is basing his work on a single diagram. She decides to start with a proportion she knows is true: $\frac{AD}{AB} = \frac{AE}{AC}$. (Why is this true?)

Tia realizes that she can rewrite this proportion as $\frac{AB+BD}{AB} = \frac{AC+CE}{AC}$ (Why is this true?)

Can you use Tia's proportion to prove algebraically that Tristan is correct?

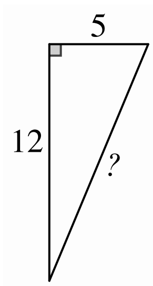
READY, SET, GO! Name Period Date

READY

Topic: Pythagorean theorem and proportions in similar triangles.

Find the missing side in each right triangle

1.



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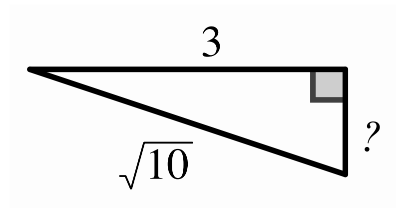
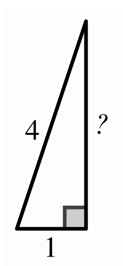


3.



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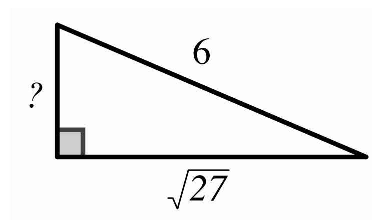
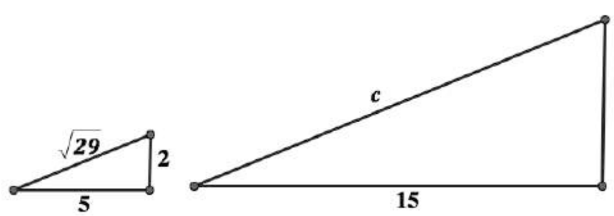


Create a proportion for each set of similar triangles. Then solve the proportion.

5.



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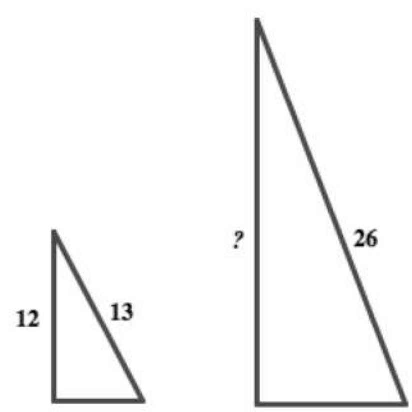

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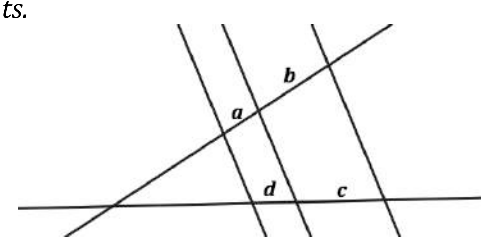
SET

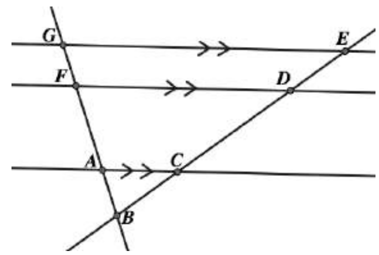
Topic: Proportionality of transversals across parallel lines

For questions 7 and 8, write three equal ratios.

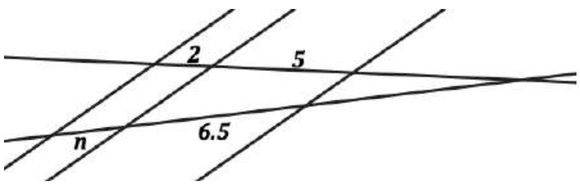
7. *The letters a,b,c and d represent lengths of line segments.*



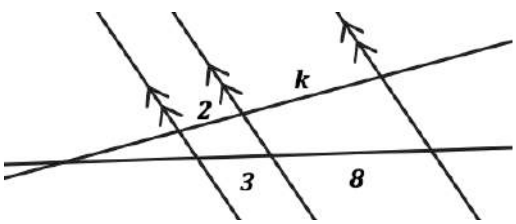
8.



9. Write and solve a proportion that will provide the missing length.

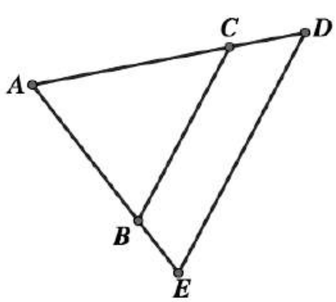


10. Write and solve a proportion that will provide the missing length.



For questions 11 – 14 find and label the parallel lines. (i.e. $\overline{AB} \parallel \overline{CD}$) Then write a similarity statement for the triangles that are similar. (i.e. $\Delta\, ABC \sim \Delta\, XYZ$)

11.



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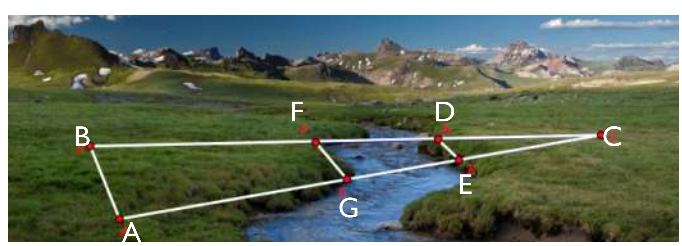

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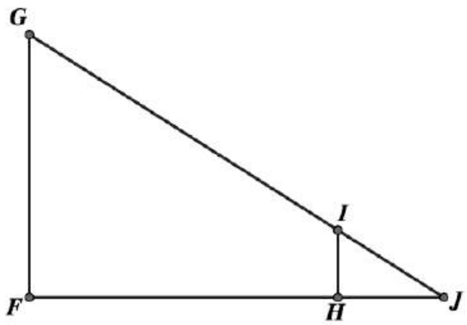
13.



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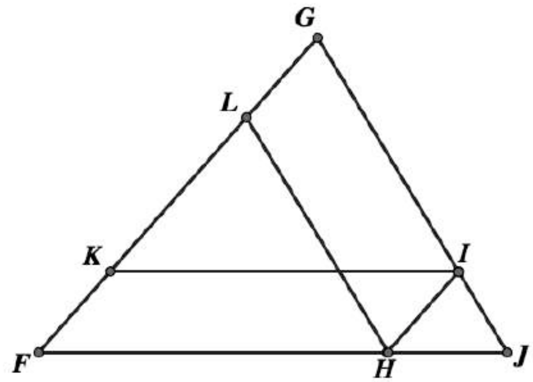
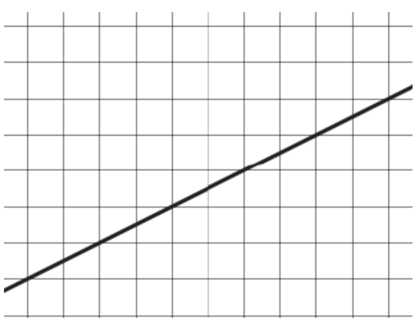


GO

Topic: Similarity in slope triangles

Each line below has several triangles that can be used to determine the slope. Draw in three slope-defining triangles of different sizes for each line and then create the ratio of rise to run for each.

15.



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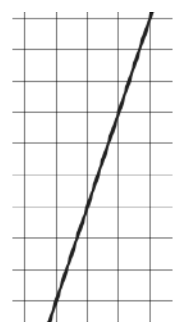
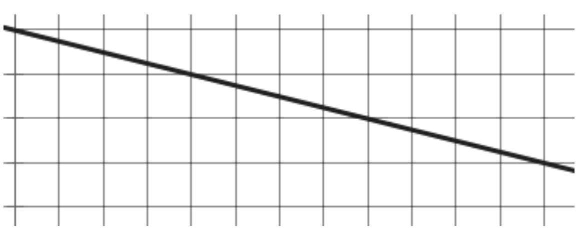
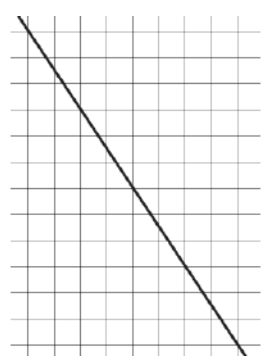


17.



18.



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6. 5 Measured Reasoning

A Practice Understanding Task

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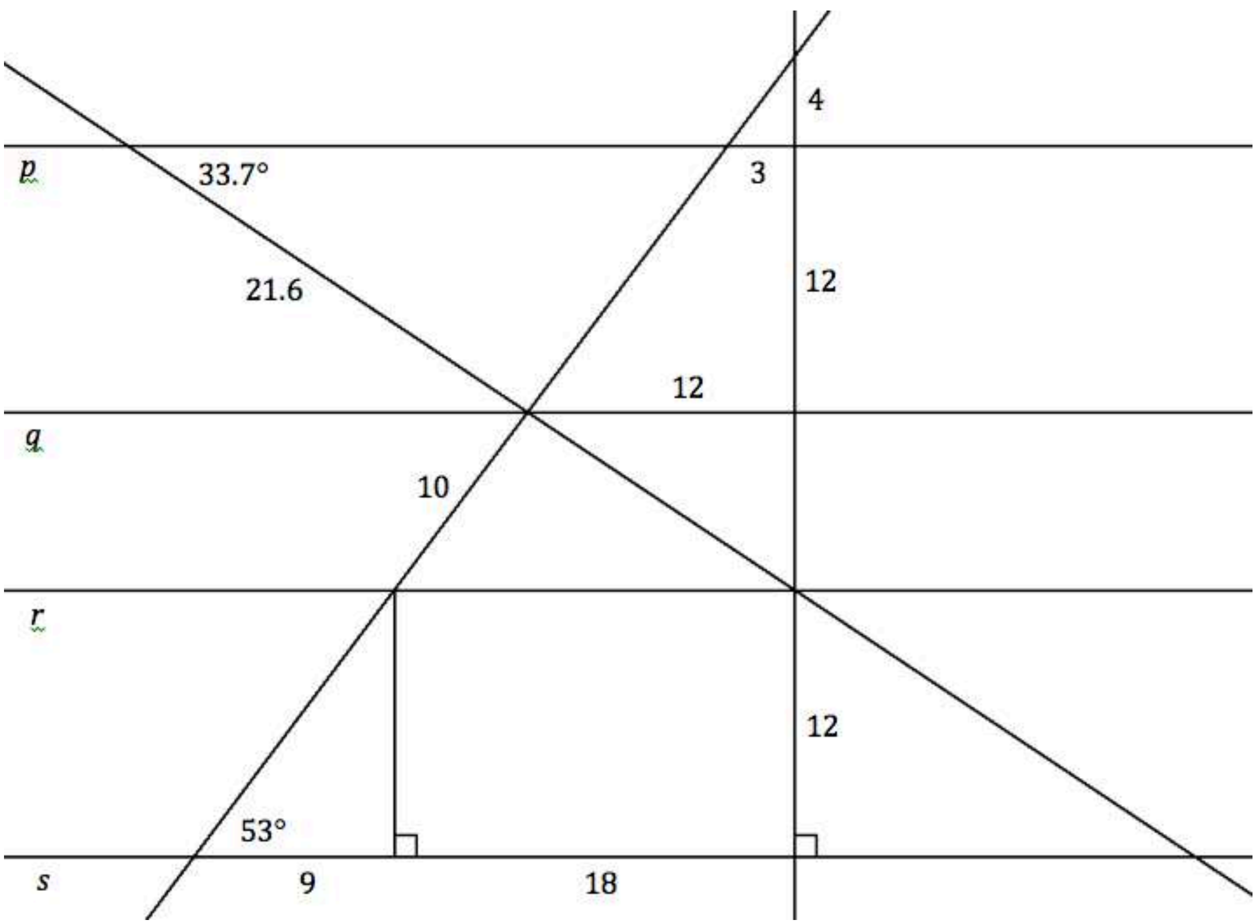
Find the measures of all missing sides and angles by using geometric reasoning, not rulers and protractors. If you think a measurement is impossible to find, identify what information you are missing.

Lines p, q, r, and s are all parallel.



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1. Identify at least three different quadrilaterals in the diagram. Find the sum of the interior angles for each quadrilateral. Make a conjecture about the sum of the interior angles of a quadrilateral.

   Conjecture:

2. Identify at least three different pentagons in the diagram. (Hint: The pentagons do not need to be convex.) Find the sum of the interior angles for each pentagon. Make a conjecture about the sum of the interior angles of a pentagon.

   Conjecture:

3. Do you see a pattern in the sum of the angles of a polygon as the number of sides increases? How can you describe this pattern symbolically?

4. How can you convince yourself that this pattern holds for all n-gons?

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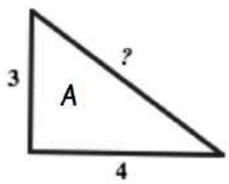
READY, SET, GO! Name Period Date

READY

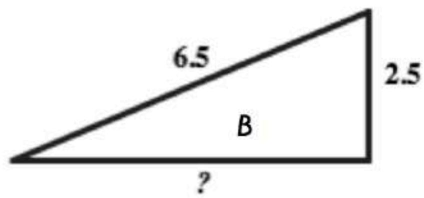
Topic: Pythagorean Theorem and ratios of similar triangles

Find the missing side in each right triangle. Triangles are not drawn to scale.

1.



2.



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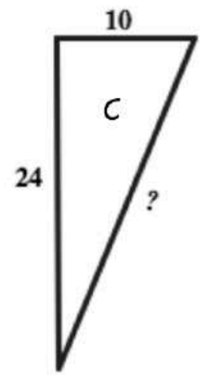

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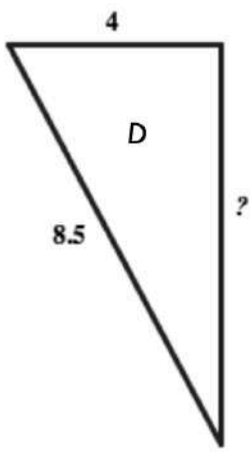

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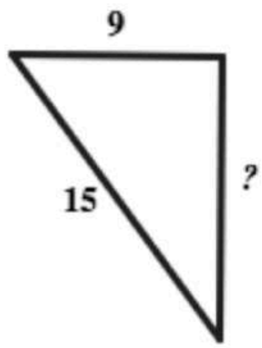

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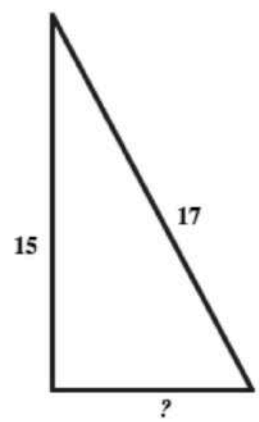

7. Based on ratios between side lengths, which of the right triangles above are mathematically similar to each other? Provide the letters of the triangles and the ratios.

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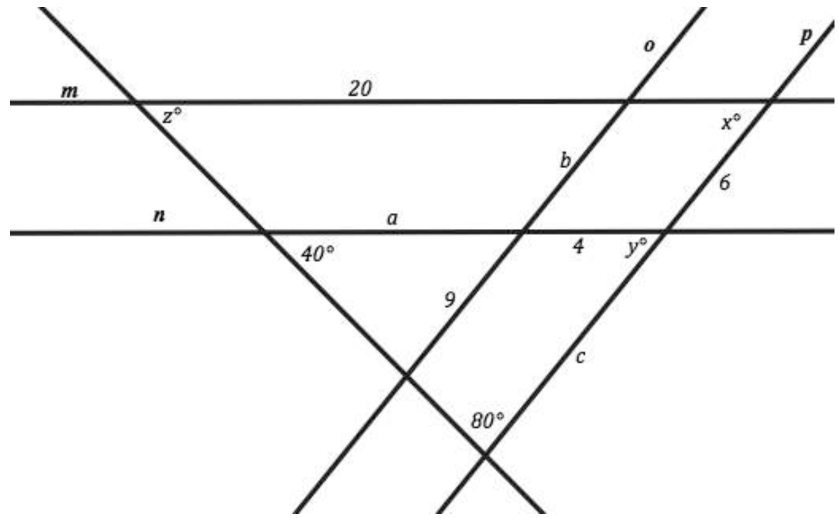
SET

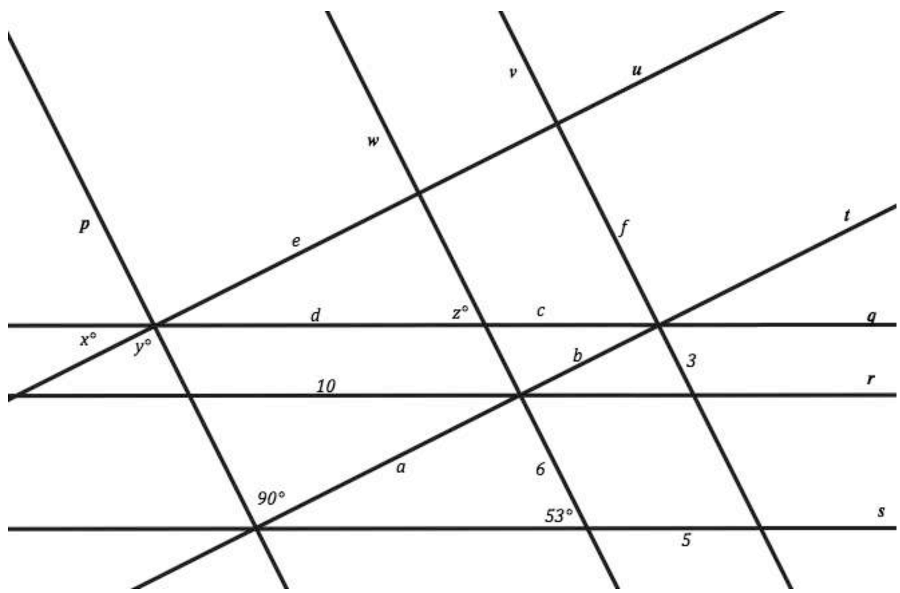
Topic: Using parallel lines, and angle relationships to find missing values.

In each of the diagrams use the given information provided to find the missing lengths and angle measurements.

8. Line $m \parallel n$ and $o \parallel p$, find the values of angles *x, y* and *z*. Also, find the lengths of *a, b* and *c*.



9. Line $q \parallel r \parallel s$ and $t \parallel u$ and $p \parallel w \parallel v$, find the values of angles *x, y* and *z*. Also, find the lengths of *a, b, c, d, e, f*.



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GO

Topic: Solve equations including those including proportions

Solve each equation below.

10.

$$3x - 5 = 2x + 7$$

11.

$$\frac{5}{7} = \frac{x}{21}$$

12.

$$\frac{3}{x} = \frac{18}{5x + 2}$$

13.

$$\frac{1}{2}x - 7 = \frac{3}{4}x - 8$$

14.

$$17 + 3(x - 5) = 2(x + 3)$$

15.

$$\frac{x + 5}{6} = \frac{3(x + 2)}{9}$$

16.

$$x + 2 + 3x - 8 = 90$$

17.

$$\frac{5}{12} = \frac{x}{8}$$

18.

$$\frac{4}{5} = \frac{x + 2}{15}$$

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6. 6 Yard Work in Segments

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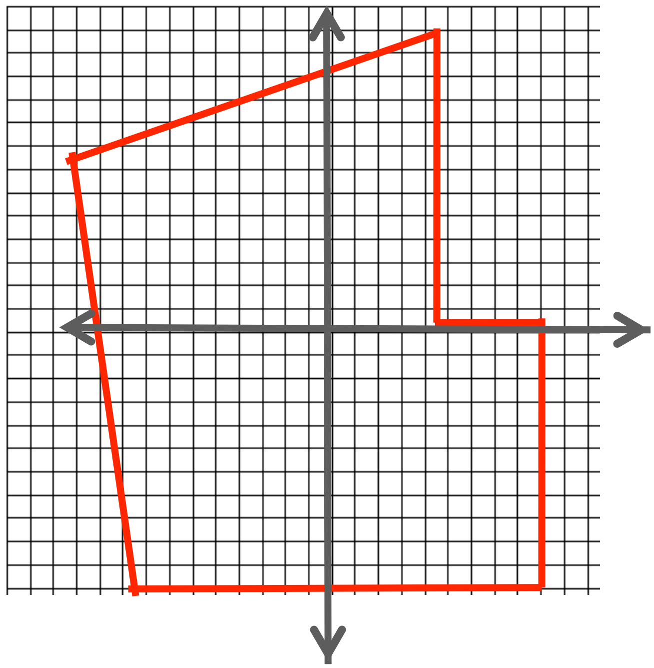
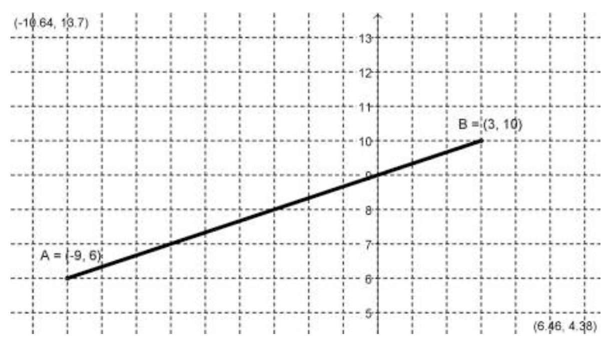
A Solidify Understanding Task

Malik's family has purchased a new house with an unfinished yard. They drew the following map of the back yard:



Malik and his family are using the map to set up gardens and patios for the yard. They plan to lay out the yard with stakes and strings so they know where to plant grass, flowers, or vegetables. They want to begin with a vegetable garden that will be parallel to the fence shown at the top of the map above.

1. They set the first stake at (-9, 6) and the stake at the end of the garden at (3, 10). They want to mark the middle of the garden with another stake. Where should the stake that is the midpoint of the segment between the two end stakes be located? Using a diagram, describe your strategy for finding this point.



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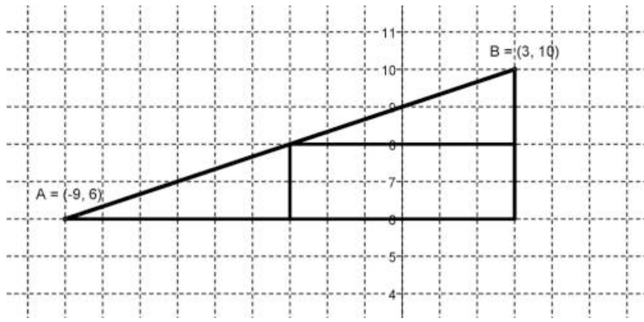
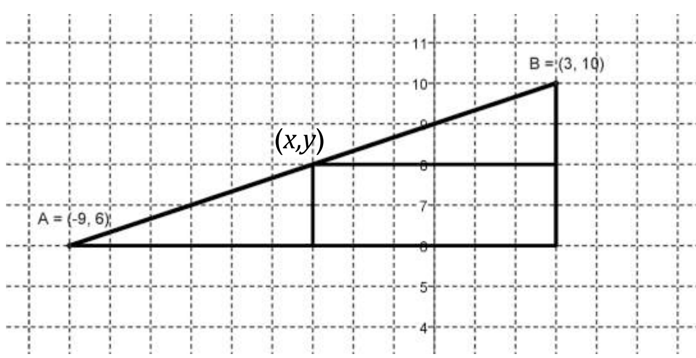
Malik figured out the midpoint by saying, "It makes sense to me that the midpoint is going to be halfway over and halfway up, so I drew a right triangle and cut the horizontal side in half and the vertical side in half like this:"



Malik continued, "That put me right at (-3, 8). The only thing that seems funny about that to me is that I know the base of the big triangle was 12 and the height of the triangle was 4, so I thought the midpoint might be (6, 2)."

2. Explain to Malik why the logic that made him think the midpoint was (6, 2) is almost right, and how to extend his thinking to use the coordinates of the endpoints to get the midpoint of (-3,6).

Malik's sister, Sapana, looked at his drawing and said, "Hey, I drew the same picture, but I noticed the two smaller triangles that were formed were congruent. Since I didn't know for sure what the midpoint was, I called it (x, y). Then I used that point to write an expression for the length of the sides of the small triangles. For instance, I figured that the base of the lower triangle was $x - (-9)$.



3. Label all of the other legs of the two smaller right triangles using Sapana's strategy.

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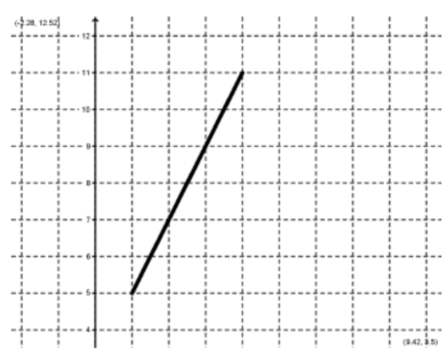
Sapana continued, "Once I labeled the triangles, I wrote equations by making the bases equal and the heights equal."

4. Does Sapana's strategy work? Show why or why not.

5. Choose a strategy and use it to find the midpoint of the segment with endpoints (-3, 4) and (2, 9).

6. Use either strategy to find the midpoint of the segment between (x_1, y_1) and (x_2, y_2).

The next area in the garden to be marked is for a flower garden. Malik's parents have the idea that part of the garden should contain a big rose bush and the rest of the garden will have smaller flowers like petunias. They want the section with the other flowers to be twice as long as the section with the rose bush. The stake on the endpoints of this garden will be at (1, 5) and (4, 11). Malik's dad says, "We'll need a stake that marks the end of the rose garden."



7. Help Malik and Sapana figure out where the stake will be located if the rose bush will be closer to the stake at (1, 5) than the stake at (4, 11).

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There's only one more set of stakes to put in. This time the endpoint stakes are at (-8, 5) and (2, -10). Another stake needs to be placed that partitions this segment into two parts so that the ratio of the lengths is 2:3.

8. Where must the stake be located if it is to be closer to the stake at (-8, 5) than to the stake at (2, -10)?

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READY, SET, GO! Name Period Date

READY

Topic: Averages, measures of center, arithmetic mean

For each set of numbers find the mean (average). Explain how the mean of the set compares to the values in the set.

1. 6, 12, 10, 8
2. 2, 7, 12
3. -13, 21
4. 3, -9, 15
5. 43, 52
6. 38, 64, 100

Find the value that is exactly half way between the two given values. Explain how you find this value.

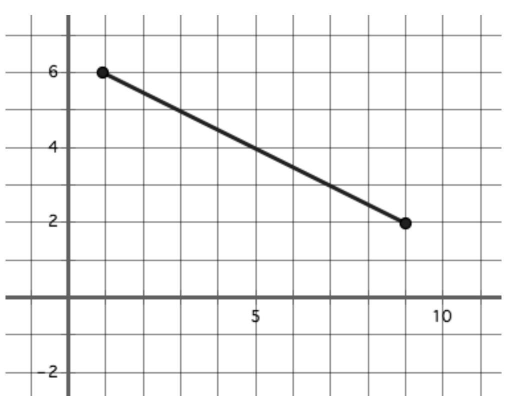
7. 5, 13
8. 26, 42
9. 57, 77
10. -34, -22
11. -45, 3
12. -12, 18

SET

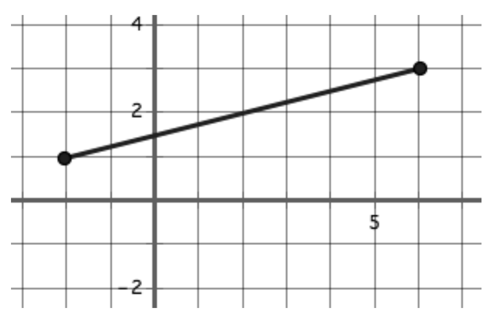
Topic: Midpoints of segments and proportionality of sides in embedded similar triangles

Find the coordinates of the midpoint of each line segment below. If multiple line segments are given then give the midpoints of all segments.

13.



14.



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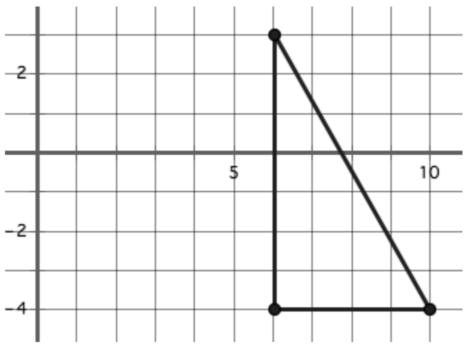
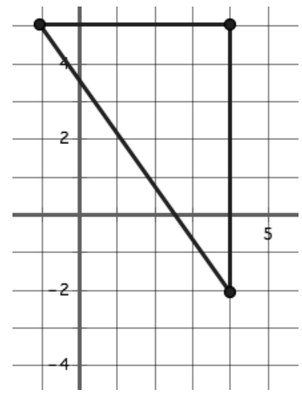
15.



16.

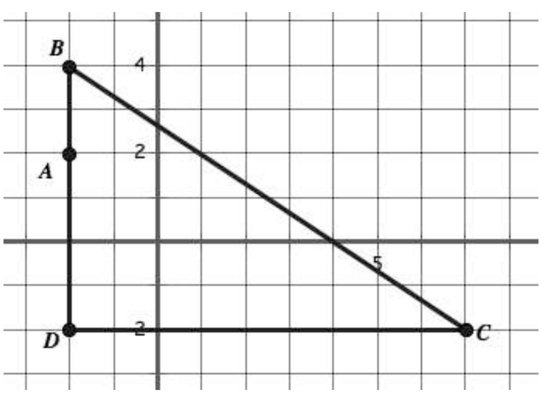


17. A line segment between (2, 3) and (10, 15)

18. A line segment between (-2, 7) and (3, -8)

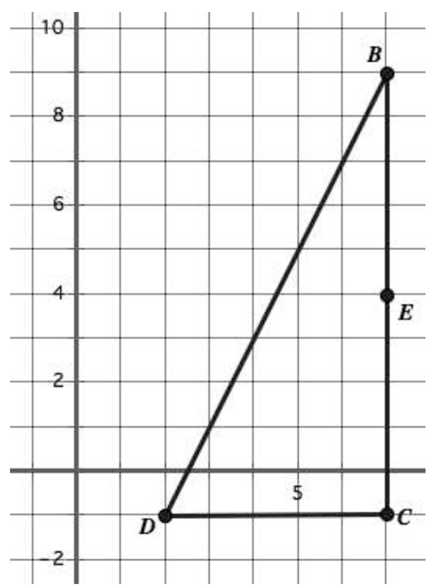
Use proportional relationships to find the desired values.

19.



If a line is drawn parallel to $\overline{BC}$ and through point A. At what coordinate will the intersection of this parallel line be with $\overline{DC}$?

20.



If a line is drawn parallel to $\overline{BD}$ and through point E. At what coordinate will the intersection of this parallel line be with $\overline{DC}$?

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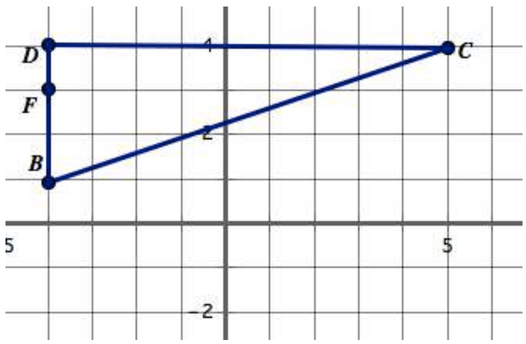
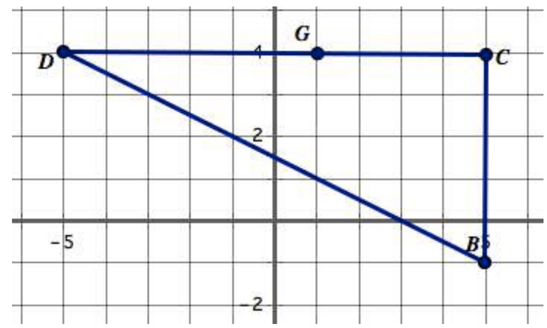
21.



If a line is drawn parallel to $\overline{BC}$ and through point F. At what coordinate will the intersection of this parallel line be with $\overline{DC}$?

22.



If a line is drawn parallel to $\overline{BD}$ and through point G. At what coordinate will the intersection of this parallel line be with $\overline{BC}$?

23. When a line is drawn parallel to one side of a triangle so that it intersects the other two sides of the triangle, how do the measures of the parts of the two intersected sides compare? Explain.

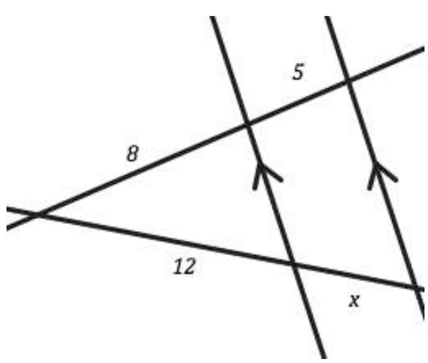
24. Problems 19-22 provided right triangles. Could a determination of the coordinates be made if they were not right triangles? Why or why not?

GO

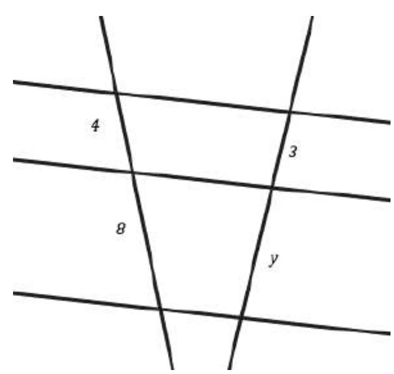
Topic: Proportionality with parallel lines.

Write a proportion for each of the diagrams below and solve for the missing value.

25.



26.



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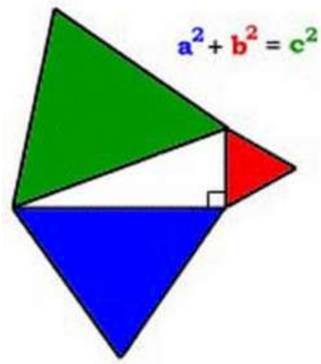
6. 7 Pythagoras by Proportions



CC BY the kirbster "Pythagorean Theorem"
https://flic.kr/p/9ho3Lr

A Practice Understanding Task

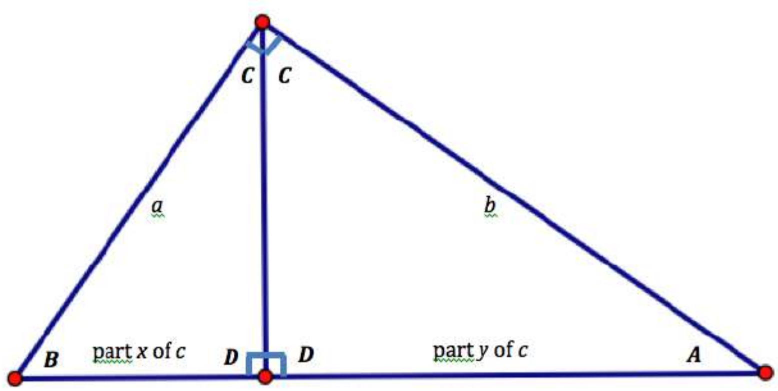
Part 1:

There are many different proofs of the Pythagorean theorem. Here is one based on similar triangles.

Step 1: Cut a 4×6 index card along one of its diagonals to form two congruent right triangles.

Step 2: In each right triangle, draw an altitude from the right angle vertex to the hypotenuse.

Step 3: Label each triangle as shown in the following diagram. Flip each triangle over and label the matching sides and angles with the same names on the back as on the front.



Step 4: Cut one of the right triangles along the altitude to form two smaller right triangles.

Step 5: Arrange the three triangles in a way that convinces you that all three right triangles are similar. You may need to reflect and/or rotate one or more triangles to form this arrangement.

Step 6: Write proportionality statements to represent relationships between the labeled sides of the triangles.

Step 7: Solve one of your proportions for x and the other proportion for y. (If you have not written proportions that involve x and y, study your set of triangles until you can do so.)

Step 8: Work with the equations you wrote in step 7 until you can show algebraically that $a^2 + b^2 = c^2$. (Remember, $x + y = c$.)

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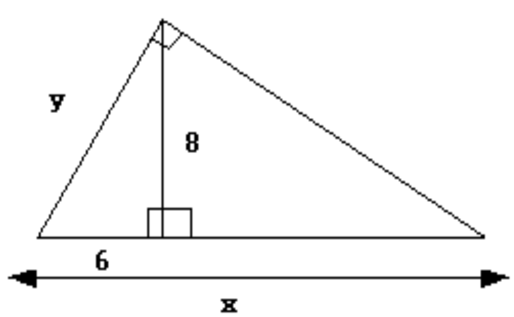
Part 2H:

Use your set of triangles to help you prove the following two theorems algebraically. For this work, you will want to label the length of the altitude of the original right triangle *h*. The appropriate legs of the smaller right triangles should also be labeled *h.*

Right Triangle Altitude Theorem 1: If an altitude is drawn to the hypotenuse of a right triangle, the length of the altitude is the geometric mean between the lengths of the two segments formed on the hypotenuse.

Right Triangle Altitude Theorem 2: If an altitude is drawn to the hypotenuse of a right triangle, the length of each leg of the right triangle is the geometric mean between the length of the hypotenuse and the length of the segment on the hypotenuse adjacent to the leg.

H: Use your set of triangles to help you find the values of *x* and *y* in the following diagram.



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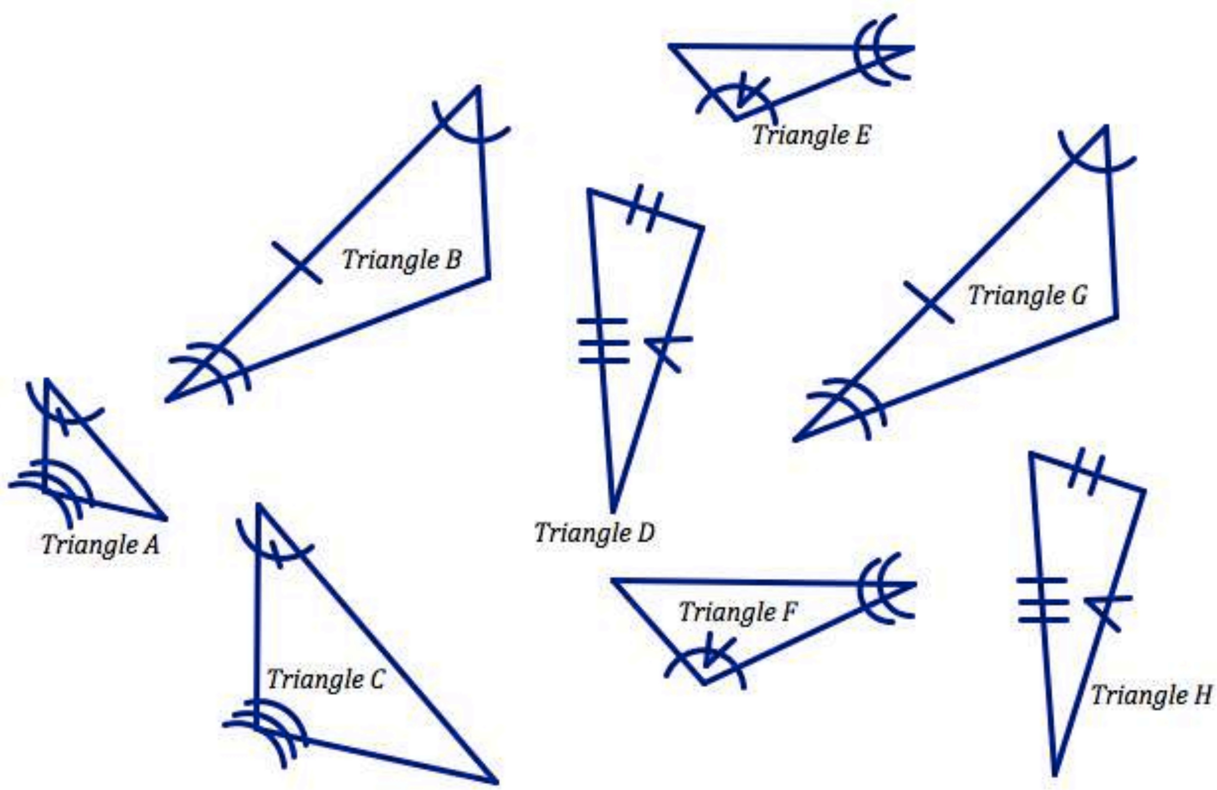
READY, SET, GO! Name Period Date

READY

Topic: Determining similarity and congruence in triangles

1. Determine which of the triangles below are similar and which are congruent. Justify your conclusions. Give your reasoning for the triangles you pick to be similar and congruent.



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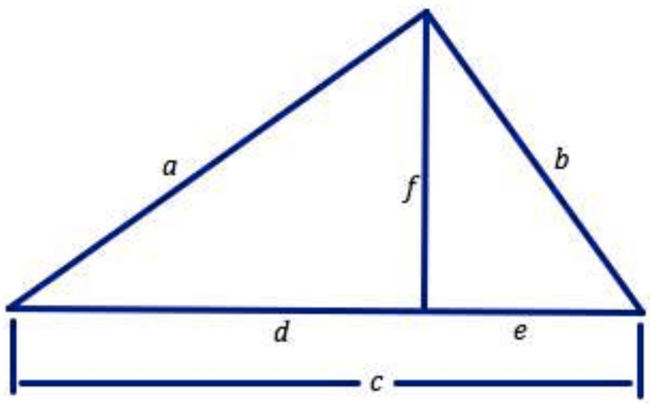
SET

Topic: Similarity in right triangles

Use the given right triangles with altitudes drawn to the hypotenuse to correctly complete the proportions.



2. $\frac{a}{d} = \frac{c}{?}$

3. $\frac{b}{c} = \frac{e}{?}$

4H. $\frac{a}{b} = \frac{f}{?}$

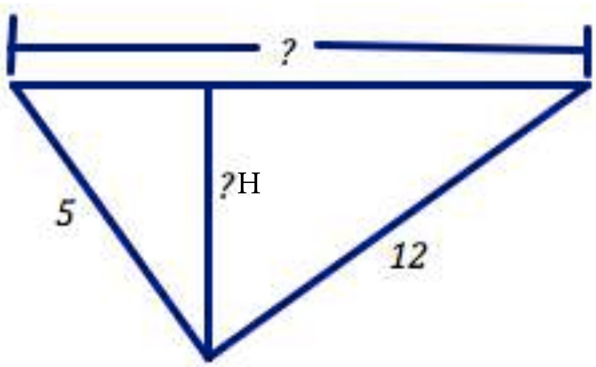
5H. $\frac{a}{c} = \frac{f}{?}$

6H. $\frac{f}{d} = \frac{e}{?}$

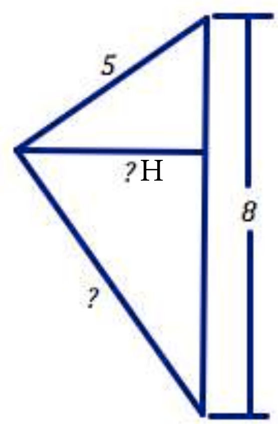
7H. $\frac{a}{f} = \frac{c}{?}$

Find the missing value for each right triangle with altitude. Honors only should find the altitude.

8.



9.



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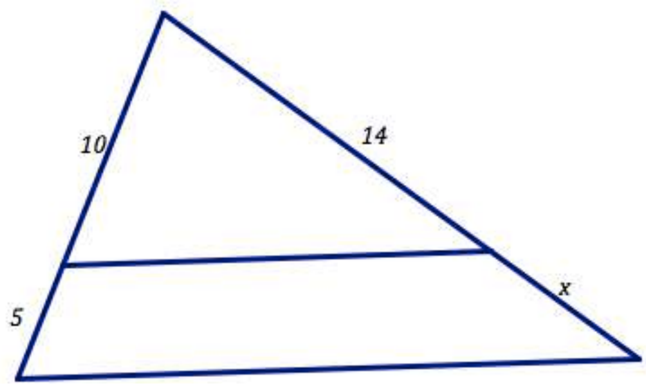
GO

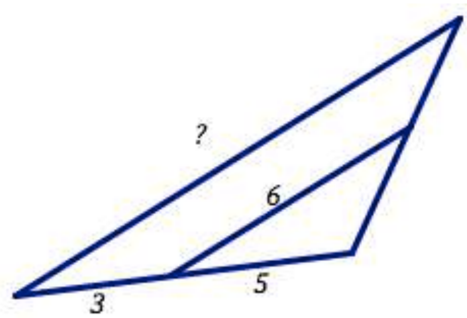
Topic: Use similarity and parallel lines to solve problems.

In each problem determine the desired values using the similar triangles parallel lines and proportional relationships. Write a proportion and solve.

10.



11.



Analyze each table below closely and determine the missing values based on the given information and values in the table.

12. An Arithmetic Sequence

Term	1	2	3	4
Value	7			22

13. A Geometric Sequence

Term	1	2	3	4
Value	7			56

14. An Arithmetic Sequence

Term	5	6	7	8
Value	10			43

15. A Geometric Sequence

Term	7	8	9	10
Value	3			24

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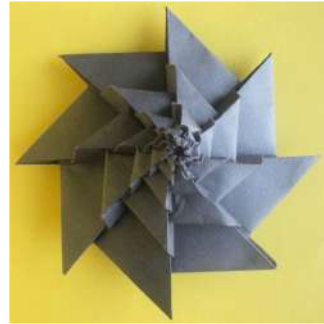
6. 8 Are Relationships Predictable?

A Develop Understanding Task



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In your notebook draw a right triangle with one angle of 60°.
Measure each side of your triangle as accurately as you can with a centimeter ruler.
Using the 60^0 angle as the ***angle of reference*** list the measure for each of the following:

Length of the ***adjacent*** side: Length of the ***opposite*** side:

Length of the ***hypotenuse***:

Create the following ratios using your measurements:

$$\frac{opposite\ side}{hypotenuse} = \qquad \frac{adjacent\ side}{hypotenuse} =$$

$$\frac{opposite\ side}{adjacent\ side} =$$

1. Compare your ratios with others that had a triangle of a different size. What do you notice? Explain any connections you find to others' work?

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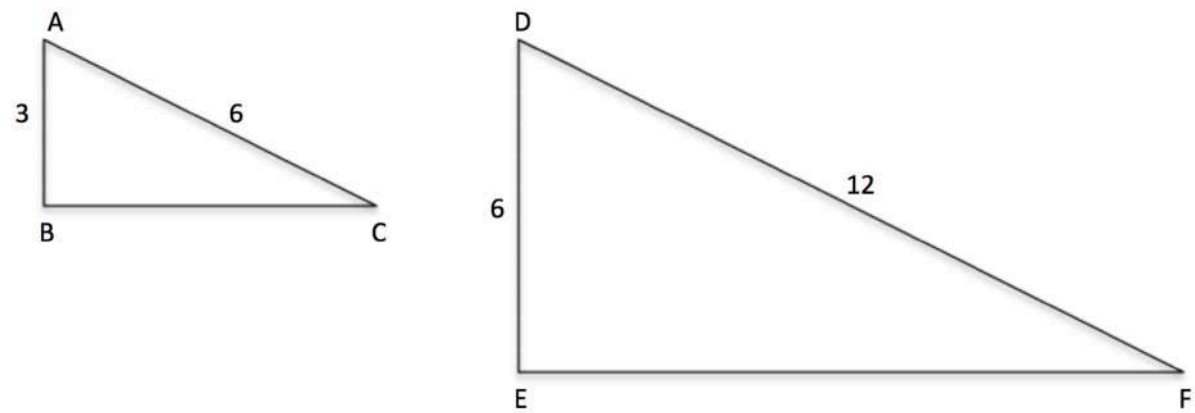
2. In the right triangles below find the missing side length and then create the desired ratios based on the angle of reference (angle A and angle D).



List the ratios for ΔABC using angle A as the angle of reference.

$$\frac{opposite\ side}{hypotenuse} =$$

$$\frac{adjacent\ side}{hypotenuse} =$$

$$\frac{opposite\ side}{adjacent\ side} =$$

List the ratios for ΔDEF using angle D as the angle of reference.

$$\frac{opposite\ side}{hypotenuse} =$$

$$\frac{adjacent\ side}{hypotenuse} =$$

$$\frac{opposite\ side}{adjacent\ side} =$$

3. What do you notice about the ratios from the two given triangles? How do these ratios compare to the ratios from the triangle you made on the previous page?

4. What can you infer about the angle measures of ΔABC and ΔDEF?

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5. Why do the relationships you have noticed occur?

6. What can you conclude about the ratio of sides in a right triangle that has a 60° angle? Would you think that right triangles with other angle measures would have similar relationships among their ratios?

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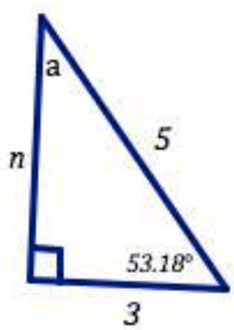
READY, SET, GO! Name Period Date

READY

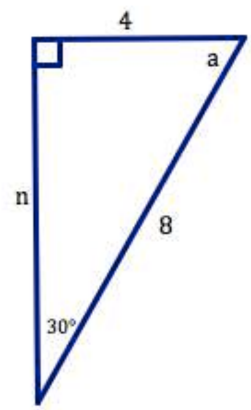
Topic: Properties of angles and sides in right triangles

For each right triangle below find the missing side *n* (Pythagorean Theorem could be helpful) and the missing angle, a (Angle Sum Theorem for Triangles could be useful).

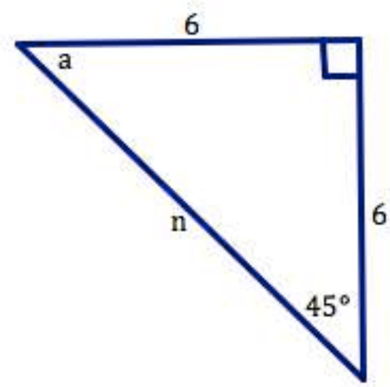
1.



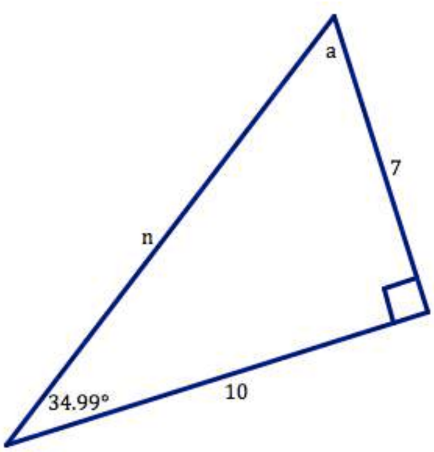
2.



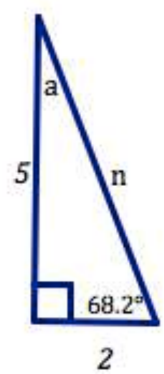
3.



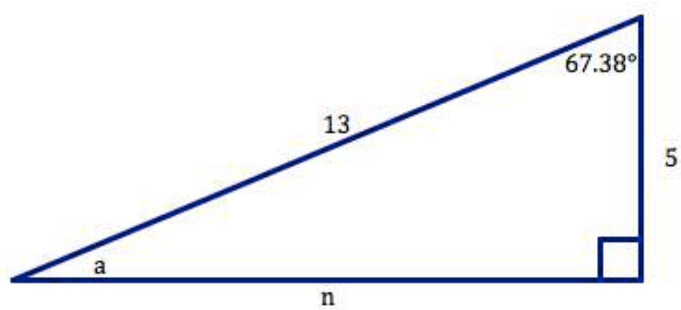
4.



5.



6.



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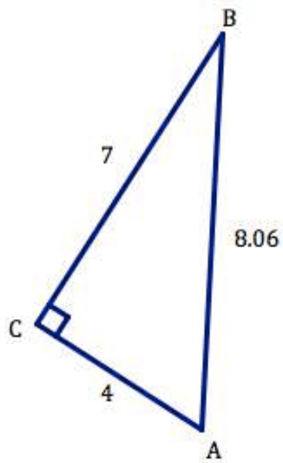
SET

Topic: Creating trigonometric ratios for right triangles

For each right triangle and the identified angle of reference create the desired trigonometric ratios. If any sides of the triangle are missing, find them before determining the ratio.

7.

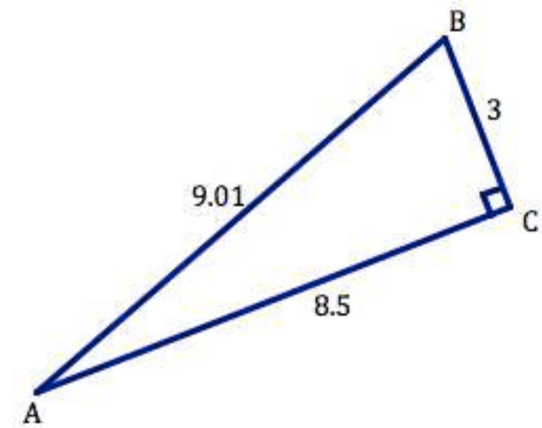


Cos (A)= Cos (B)=

Sin (A)= Sin (B)=

Tan (A)= Tan (B)=

8.

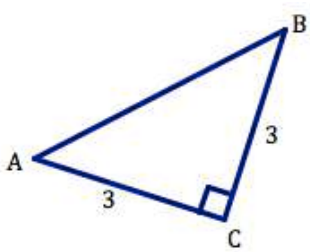


Cos (A)= Cos (B)=

Sin (A)= Sin (B)=

Tan (A)= Tan (B)=

9.

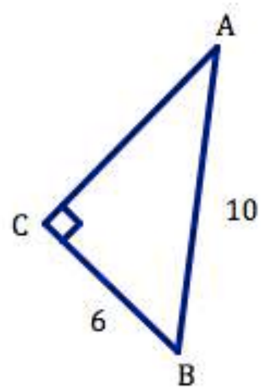


Cos (A)= Cos (B)=

Sin (A)= Sin (B)=

Tan (A)= Tan (B)=

10.



Cos (A)= Cos (B)=

Sin (A)= Sin (B)=

Tan (A)= Tan (B)=

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GO

Topic: Factoring quadratics

Provide the factored form and the x- and y-intercepts

11. $f(x)= x^2 + 9x + 20$

factored form:

x-intercepts:

y-intercept:

12. $g(x)= x^2 + 2x - 15$

factored form:

x-intercepts:

y-intercept:

13. $h(x)= x^2 - 49$

factored form:

x-intercepts:

y-intercept:

14. $r(x)= x^2 - 13x + 30$

factored form:

x-intercepts:

y-intercept:

15. $f(x)= x^2 + 20x + 100$

factored form:

x-intercepts:

y-intercept:

16. $g(x)= x^2 - 8x - 48$

factored form:

x-intercepts:

y-intercept:

17. $h(x)= x^2 + 16x + 64$

factored form:

x-intercepts:

y-intercept:

18. $k(x)= x^2 - 36$

factored form:

x-intercepts:

y-intercept:

19. $p(x)= x^2 - 2x - 24$

factored form:

x-intercepts:

y-intercept:

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6. 10 Finding the Value of a Relationship


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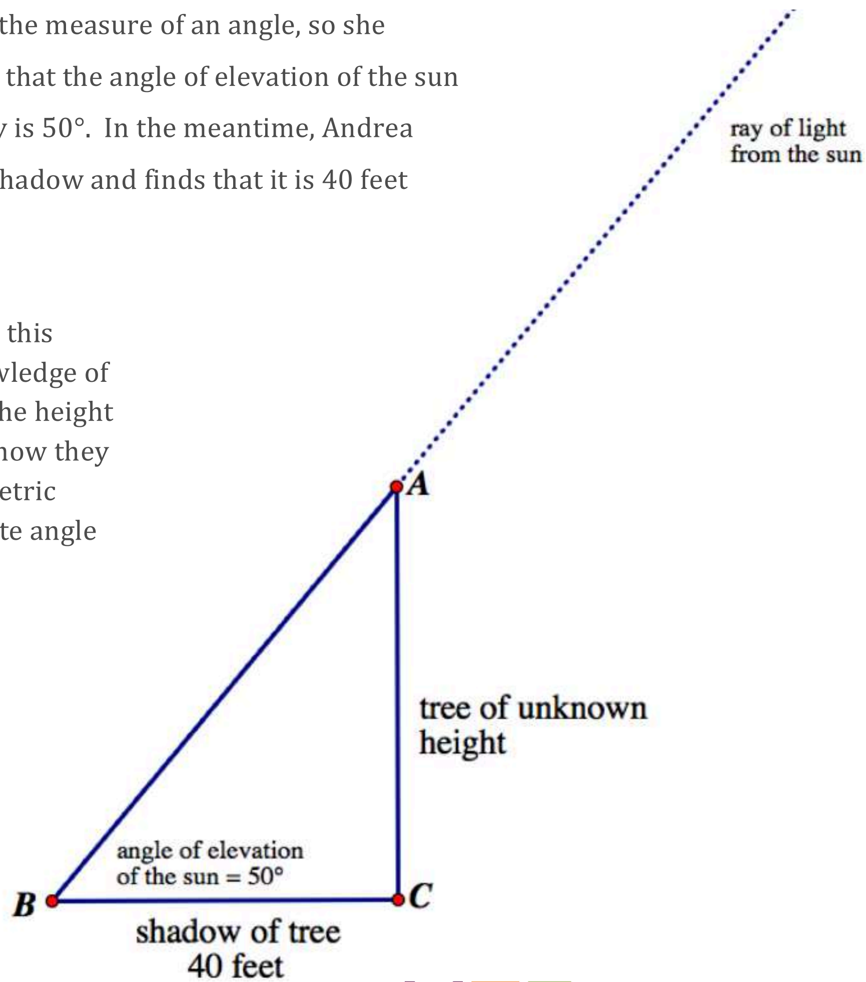
A Solidify Understanding Task

Part 1: Pick a side

Andrea and Bonita are resting under their favorite tree before taking a nature walk up a hill. Both girls have been studying trigonometry in school, and now it seems like they see right triangles everywhere. For example, Andrea notices the length of the shadow of the tree they are sitting under and wonders if they can calculate the height of the tree just by measuring the length of its shadow.

Bonita thinks they also need to know the measure of an angle, so she checks an app on her phone and finds that the angle of elevation of the sun at the current location and time of day is 50°. In the meantime, Andrea has paced off the length of the tree's shadow and finds that it is 40 feet long.

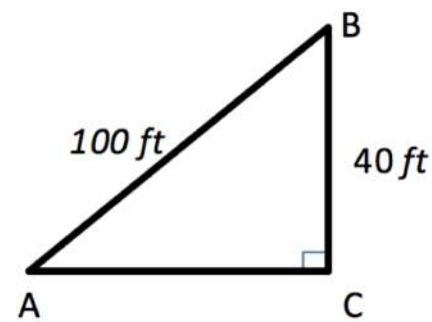
1. How might Andrea and Bonita use this information, along with their knowledge of trigonometric ratios, to calculate the height of the tree? (Andrea and Bonita know they can find the value of any trigonometric ratios they might need for any acute angle using a calculator.)



Part 2: What's your angle?

After their rest, Andrea and Bonita are going for a walk straight up the side of the hill. Andrea decided to stretch before heading up the hill while Bonita thought this would be a good time to get a head start. Once Bonita was 100 feet away from Andrea, she stopped to take a break and looked at her GPS device that told her that she had walked 100 feet and had already increased her elevation by 40 feet. With a bit of time to waste, Bonita wrote down the trigonometric ratios for ∠A and for ∠B.

2. Name the trigonometric ratios for ∠A and for ∠B.



When Andrea caught up, she said "What about the unknown angle measures? When I was at the bottom and looked up to see you, I was thinking about the "upward" angle measure from me to you. Based on your picture, this would be ∠A."

Bonita wrote the trigonometric ratio $\sin A = \frac{40}{100}$ and asked, "So, how do we find angle A?"

Together, the girls talked about how this was like thinking backwards: instead of knowing an angle and using their calculators to find a trigonometric ratio like they did while working on the height of the tree problem, they now know the trigonometric ratio and need to find an unknown angle value. Bonita notices the $sin^{-1}(\theta)$ button on her calculator and wonders if this might work like an "inverse trigonometric ratio" button, undoing the ratio to produce the angle. She decides to try it out, and produces the following output on her calculator:

$\sin A = \frac{40}{100}$

$sin^{-1}\left(\frac{40}{100}\right) = 23.578°$

$\sin(23.578°) = 0.4$

3. How might this output convince Bonita that her assumption about the calculator was correct?

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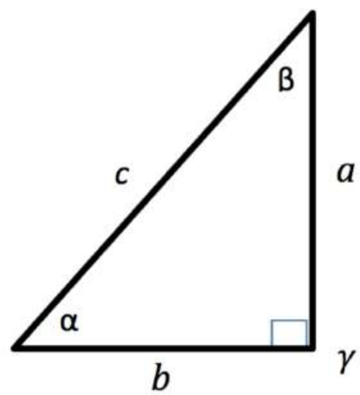
4. Use the trigonometric ratio you found for $\cos B$ to find the value of $\angle B$.

5. Find all unknown values for the following right triangle:



   a) $\angle\alpha =$ _____
   b) $\angle\beta =$ _____
   c) $\angle\gamma =$ <u>90°</u>
   d) $a =$ <u>*12 m*</u>
   e) $b =$ <u>*8 m*</u>
   f) $c =$ _____

6. Bonita and Andrea started talking about all of the ways to find unknown values in right triangles and decided to make a list. What do you think should be on their list? Be specific and precise in your description. For example, 'trig ratios' is not specific enough. You may use the following sentence frame to assist with writing each item in your list:

When given ____________________, you can find ____________________ by _______________.

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Part 3: Angle of elevation and angle of depression

During their hike, Andrea mentioned that she looked up to see Bonita. In mathematics, when you look straight ahead, we say your line of sight is a horizontal line. From the horizontal, if you look up, the angle from the horizontal to your line of sight is called the **angle of elevation.** Likewise, if you are looking down, the angle from the horizontal to your line of sight is called the **angle of depression.**

7. After looking at this description, Andrea mentioned that her angle of elevation to see Bonita was about 23.5°. They both agreed. Bonita then said her angle of depression to Andrea was about 66.5°. Andrea agreed that it was an angle of depression but said Bonita's angle of depression was also 23.5°. Who do you think is correct? Use drawings and words to justify your conclusion.

8. What conclusion can you make regarding the angle of depression and the angle of elevation? Why?

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READY, SET, GO! Name Period Date

READY

Topic: Modeling real world problems with triangles.

For each story presented below *sketch a picture* of the situation *and label as much of the picture as possible.* No need to answer the question or find the missing values, simply represent the situation with a sketch.

1. Jill put a ladder up against the house to try and reach a light that is out and needs to be changed. She knows the ladder is 10 feet long and the distance from the base of the house to the bottom of the ladder is 4 feet.

2. Francis is a pilot of an airplane that if flying at an altitude of 3,000 feet when the plane begins its descent toward the ground. If the angle of decent of the plane is 15^0 how much farther will the plane fly before it is on the ground?

3. Abby is standing at the top of a very tall skyscraper and looking through a telescope at the scenery all around her. The angle of decline on the telescope says 35^0 and Abby knows she is 30 floors up and each floor is 15 feet tall. How far from the base of the building is the object that Abby is looking at?

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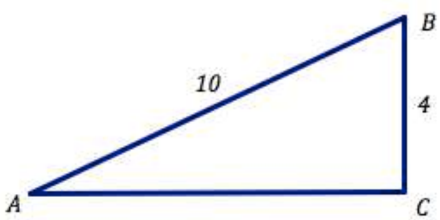
SET

Topic: Solving triangles using Trigonometric Ratios

In each triangle find the missing angles and sides.

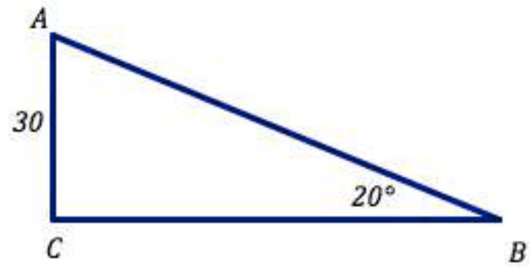
4.



$m\angle A =$ $m\angle B =$

$m\angle C = 90^o$ $AC =$

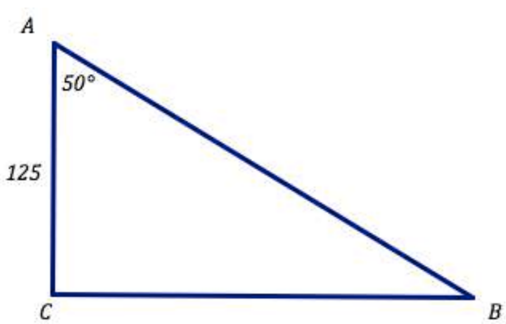
5.



$m\angle A =$ $AB =$

$m\angle C = 90^o$ $BC =$

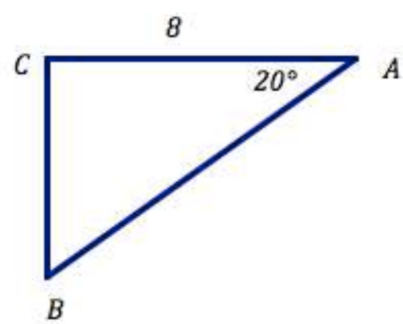
6.



$AB =$ $m\angle B =$

$m\angle C = 90^o$ $BC =$

7.



$AB =$ $m\angle B =$

$m\angle C = 90^o$ $BC =$

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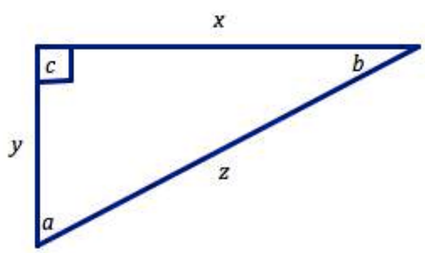
GO

Topic: Trigonometric ratios

Use the given right triangle to identify the trigonometric ratios. And angles were possible.

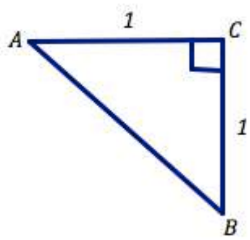
8.



$\sin(a) =$ $\cos(a) =$ $\tan(a) =$

$\sin(b) =$ $\cos(b) =$ $\tan(b) =$

9.

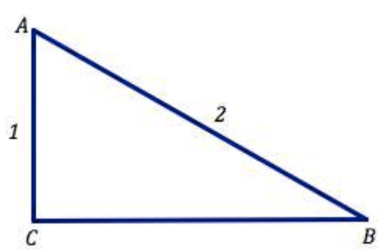


$\sin(A) =$ $\cos(A) =$ $\tan(A) =$

$\sin(B) =$ $\cos(B) =$ $\tan(B) =$

m∠A= m∠B=

10.



$\sin(A) =$ $\cos(A) =$ $\tan(A) =$

$\sin(B) =$ $\cos(B) =$ $\tan(B) =$

m∠A= m∠B= m∠C= 90°

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5.5 Special Rights

A Solidify Understanding Task

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In previous courses you have studied the Pythagorean theorem and right triangle trigonometry. Both of these mathematical tools are useful when trying to find missing sides of a right triangle.

1. What do you need to know about a right triangle in order to use the Pythagorean theorem?

2. What do you need to know about a right triangle in order to use right triangle trigonometry?

While using the Pythagorean theorem is fairly straight forward (you only have to keep track of the legs and hypotenuse of the triangle), right triangle trigonometry generally requires a calculator to look up values of different trig ratios. There are some right triangles, however, for which knowing a side length and an angle is enough to calculate the value of the other sides without using trigonometry. These are known as *special right triangles* because their side lengths can be found by relating them to another geometric figure for which we know something about its sides.

<u>One type of special right triangle is a 45°-45°-90° triangle</u>.

3. Draw a 45°-45°-90° triangle and assign a specific value to one of its sides. (For example, let one of the legs measure 5 cm, or choose to let the hypotenuse measure 8 inches. You will want to try both approaches to perfect your strategy.) Now that you have assigned a measurement to one of the sides of your triangle, find a way to calculate the measures of the other two sides. As part of your strategy, you may want to relate this triangle to another geometric figure that may be easier to think about.

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4. Generalize your strategy by letting one side of the triangle measure *x*. Show how the measure of the other two sides can be represented in terms of *x*. (Make sure to consider cases where *x* is the length of a leg, as well as the case where *x* is the length of the hypotenuse.)

Another type of special right triangle is a 30°-60°-90° triangle.

5. Draw a 30°-60°-90° triangle and assign a specific value to one of its sides. Now that you have assigned a measurement to one of the sides of your triangle, find a way to calculate the measures of the other two sides. As part of your strategy, you may want to relate this triangle to another geometric figure that may be easier to think about.

6. Generalize your strategy by letting one side of the triangle measure *x*. Show how the measure of the other two sides can be represented in terms of *x*. (Make sure to consider cases where *x* is the length of a leg, as well as the case where *x* is the length of the hypotenuse.)

7. Can you think of any other angle measurements that will create a special right triangle?

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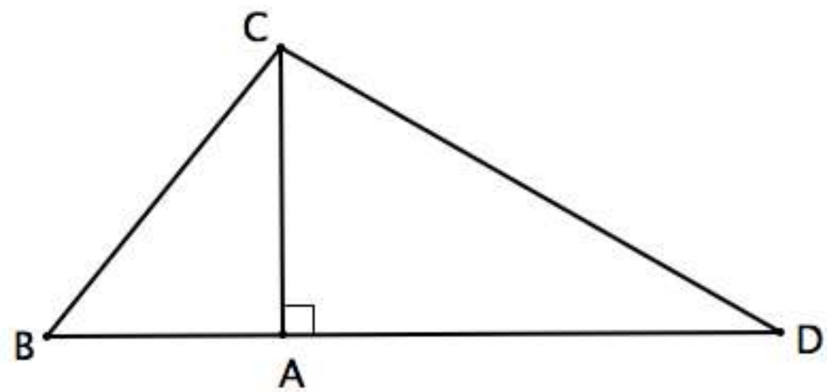
READY, SET, GO! Name Period Date

READY

Topic: Finding missing measures in triangles

Use the given figure to answer the questions. Round your answers to the hundredths place.

Given: $m\angle CBD = 51°$
$m\angle CDA = 30°$



1. Find $m\angle BCD$

Given : $m\angle CAD = 90°$

2. Find $m\angle BCA$ and $m\angle ACD$

Given: ***CA = 6 ft***

3. Find BC

4. Find BA

5. Find CD

6. Find AD

7. Find BD

8. Find the area of $\Delta\, BCD$

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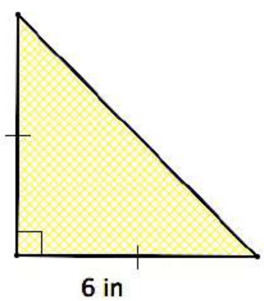
SET

Topic: Recalling triangle relationships in Special Right Triangles

Fill in all the missing measures in the triangles.

9.



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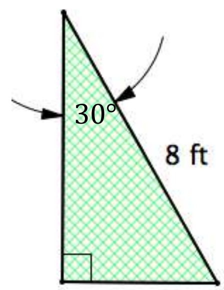

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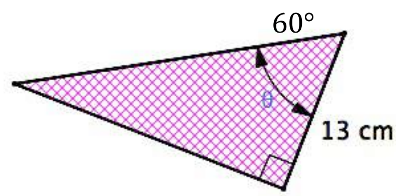
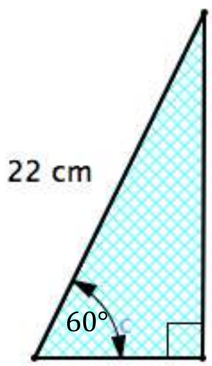


12.



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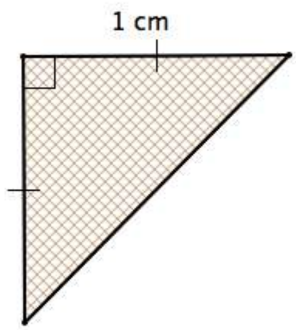

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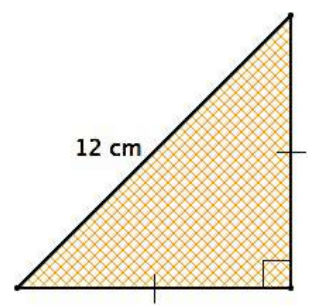

Use an appropriate triangle from above to fill in the function values below. No calculators.

15.

$\sin 45° =$	
$\cos 45° =$	
$\tan 45° =$	

16.

$\sin 30° =$	
$\cos 30° =$	
$\tan 30° =$	

17.

$\sin 60° =$	
$\cos 60° =$	
$\tan 60° =$	

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GO

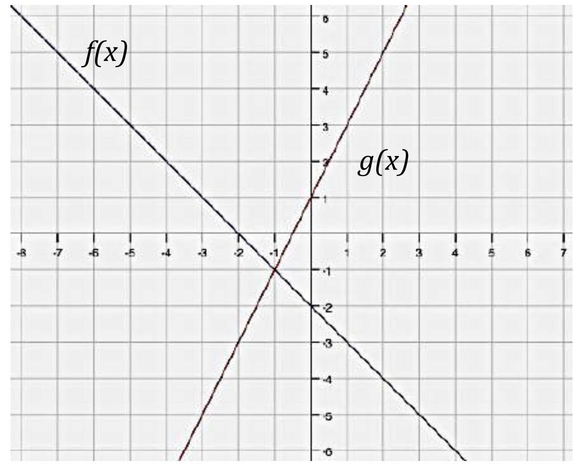
Topic: Performing function arithmetic on a graph

18. Add $f(x)$ and $g(x)$ using the graph at the right.

Draw the new figure on the graph and label it as *s(x), the sum of x.*

19. Subtract $f(x)$ from $g(x)$ using the graph at the right.

Draw the new figure on the graph and label it as *d(x), the difference of x.*

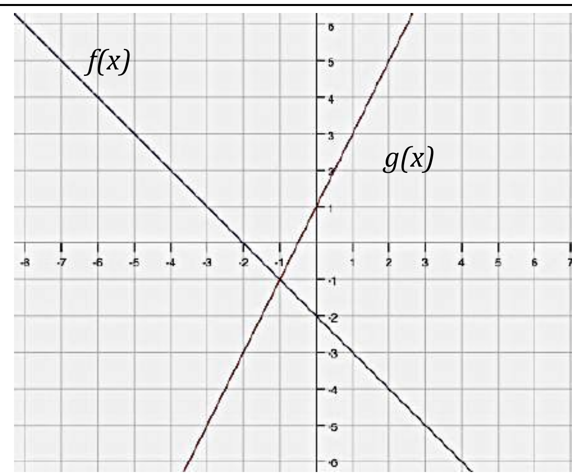


20. Multiply $f(x)$ and $g(x)$ on the second graph at the right.

Draw the new figure on the graph and label it as *p(x), the product of x.*

21. Divide $f(x)$ by $g(x)$ on the second graph at the right.

Draw the new figure on the graph and label it as *q(x), the quotient of x.*



22. Write the equations of $f(x)$ and $g(x)$.	
23. Write the equation of the sum of $f(x)$ and $g(x)$. $s(x) =$	24. Write the equation of the difference of $f(x)$ and $g(x)$. $d(x) =$
25. Write the equation of the product of $f(x)$ and $g(x)$. $p(x) =$	26. Write the equation of the quotient of $f(x)$ divided by $g(x)$. $q(x) =$

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6. 11 Solving Right Triangles Using Trigonometric Relationships

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A Practice Understanding Task

I. For each problem:

- ***make a drawing***
- ***write an equation***
- ***solve (do not forget to include units of measure)***

1. Carrie places a 10-foot ladder against a wall. If the ladder makes an angle of 65° with the level ground, how far up the wall is the top of the ladder?

2. A flagpole casts a shadow that is 15 feet long. The angle of elevation of the sun at this time is 40°. How tall is the flagpole?

3. In southern California, there is a six-mile section of Interstate 5 that increases 2,500 feet in elevation. What is the angle of elevation?

4. A hot air balloon is 100 feet straight above where it is planning to land. Sarah is driving to meet the balloon when it lands. If the angle of elevation to the balloon is 35°, how far away is Sarah from place on the ground where the balloon will land?

5. An airplane is descending as it approaches the airport. If the angle of depression from the plane to the ground is 7°, and the plane is 2,000 feet above the ground, what is the distance from the plane to the airport?

6. Michelle is 60 feet away from a building. The angle of elevation to the top of the building is 41°. How tall is the building?

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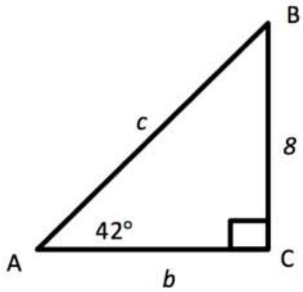
7. A ramp is used for loading equipment from a dock to a ship. The ramp is 10 feet long and the ship is 6 feet higher than the dock. What is the angle of elevation of the ramp?

II. For each right triangle below, find all unknown side lengths and angle measures:

8.



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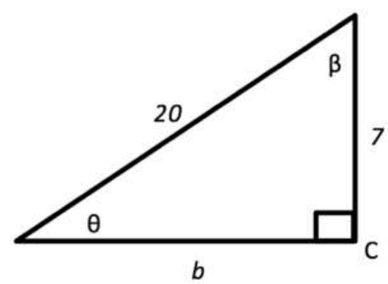

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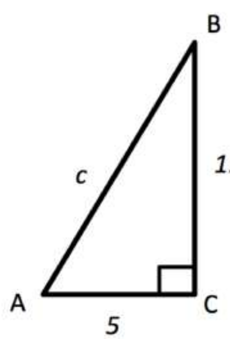

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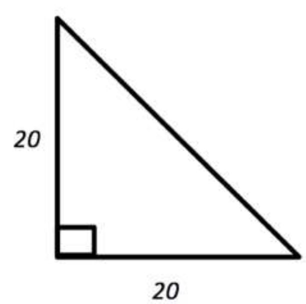

12. Draw and find the missing angle measures of the right triangle whose sides measure 4, 6, and 8.

III. Determine the values of the two remaining trigonometric ratios when given one of the trigonometric ratios.

13. $\cos(\alpha) = \frac{3}{5}$

14. $\tan(\theta) = \frac{8}{3}$

15. $\sin(\beta) = \frac{4}{7}$

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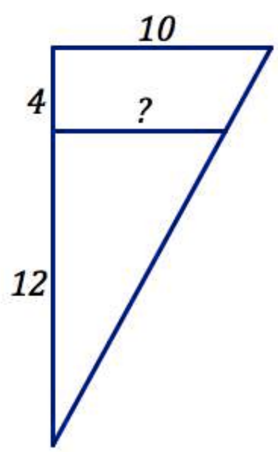
READY, SET, GO! Name Period Date

READY

Topic: Similar triangles and proportional relationships with parallel lines

Based on each set of triangles or parallel lines create a proportion and solve it to find the missing values.

1.



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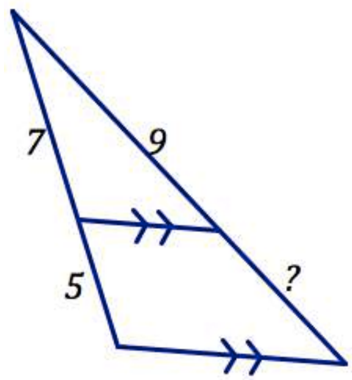
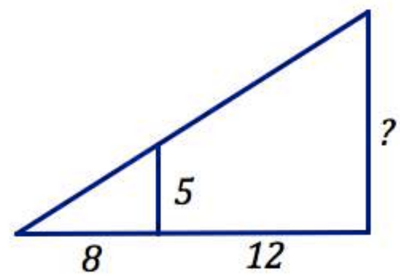


3.



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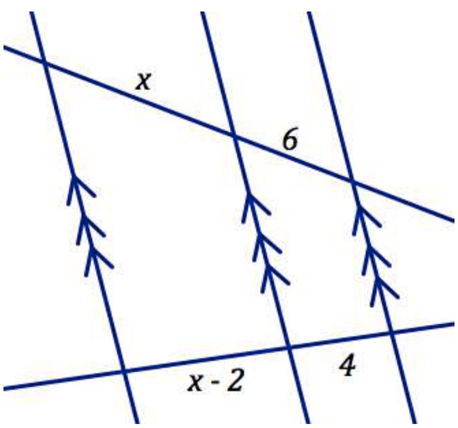

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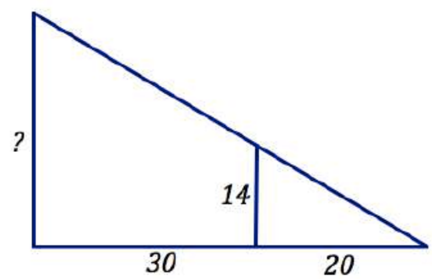

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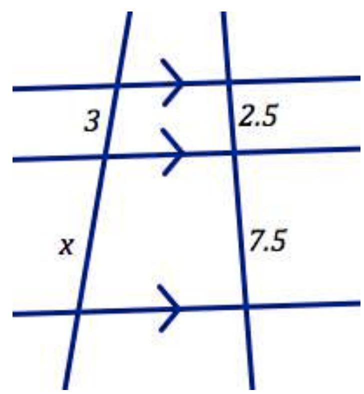

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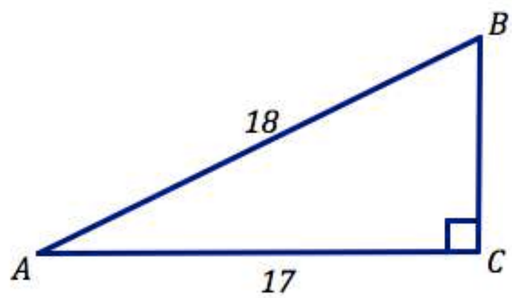
SET

Topic: Solving triangles with trigonometric ratios and Pythagorean theorem

Solve each right triangle. Give any missing sides and missing angles.

7.



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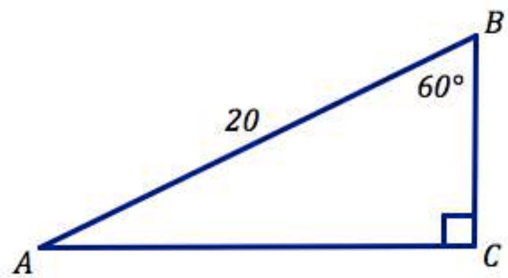

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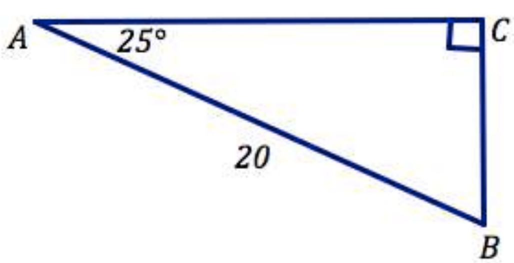

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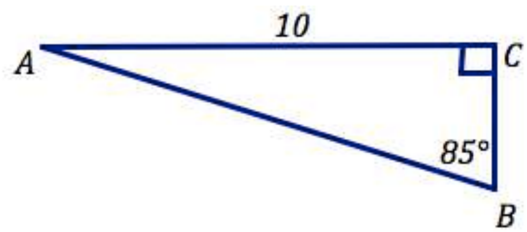

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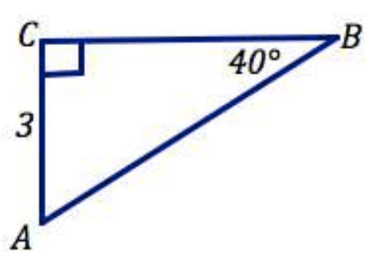

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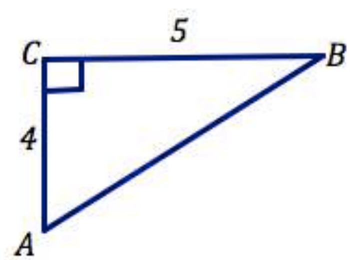

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Use the given trigonometric ratio to sketch a right triangle and solve the triangle.

13. $\sin(A) = \frac{1}{2}$

14. $\cos(B) = \frac{3}{5}$

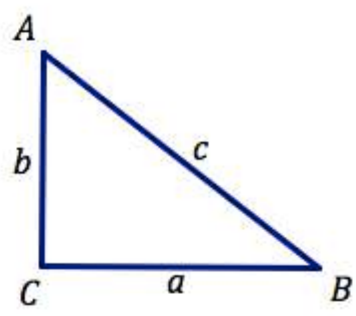
15. $\tan(B) = \frac{6}{7}$

16. $\sin(B) = \frac{7}{10}$

17. $\cos(A) = \frac{5}{8}$

18. $\tan(A) = \frac{4}{15}$

Use the right triangle below to determine which of the following are equivalent.



19. $\sin(A)$
20. $\cos(A)$
21. $\tan(A)$
22. $\sin(B)$
23. $\cos(B)$
24. $\tan(B)$
25. $\frac{\sin(A)}{\cos(A)}$
26. $\frac{1}{\tan(A)}$
27. 1
28. $a^2 + b^2$
29. c^2
30. $\sin^2(B)+\cos^2(B)$

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GO

Topic: Applying trigonometric ratios and identities to solve problems.

Solve each problem. Sketch a drawing of the situation.

31. Mark is building his son a pitcher's mound so he can practice for his upcoming baseball season in the back yard. Mark knows that the league requires an incline of 12° and an elevation of 8 inches in height. How long will the front of the pitcher's mound need to be?

32. Susan is designing a wheelchair ramp. Wheelchair ramps require a slope that is no more than 1-inch of rise for every 12-inches of ramp length. Susan wants to determine how much horizontal distance a ramp of 6-feet in length will span? She also wants to know the degree of incline from the base of the ramp to the ground.

33. Michael is designing a house with a roof pitch of 5. Roof pitch is the number of inches that a roof will rise for every 12 inches of run. What is the angle that will need to be used in building the trusses and supports for the roof? What is the angle of a roof with 5/12 pitch increase? At the peak of the roof what angle will there be when the front and the back of the roof come together?

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